Information in neutron interference experiments

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Abstract

Wigner measures are calculated for a number of neutron interference experiments that can be interpreted as joint nonideal measurements of path and interference. It is demonstrated that the Wigner measure need not be influenced by fluctuations, thus showing that fluctuations need not diminish the information content of the experiment. A measurement is proposed yielding complete information on the incoming neutron state.

1 Introduction

Complementarity of interference and path measurements in quantum mechanics can be studied in neutron interferometry by inserting an absorber (transmission probability $a, 0 < a < 1$) in one of the internal interferometer paths. Such measurements were performed experimentally [1], and discussed theoretically [2], using the Davies-Ludwig generalization of the quantum mechanical formalism, as joint nonideal measurements of quantum mechanical interference and path observables. In the Davies-Ludwig formalism quantum mechanical observables are not generally represented by hermitian operators but by positive operator valued measures (POVM), the projection valued measures of the usual Dirac-von Neumann formalism being special cases.

A measurement of the type discussed here is interesting because its POVM yields simultaneous quantum mechanical information on two incompatible Dirac-von Neumann observables, viz., the interference and the path observable. Compared with a measurement of either the interference observable ($a = 1$) or the path observable ($a = 0$) the information obtained in the joint measurement ($0 < a < 1$) is nonideal [2, 3], the nonideality being interpretable in the sense of complementarity, namely as a mutual disturbance in the joint measurement of two incompatible observables. For the interference observable this nonideality can, for instance, be observed as a reduction of the visibility of the interference pattern of one of the neutron detectors, which can be calculated [4] to be

$$V_{\mathbf{M}_A} = \frac{2\sqrt{a}}{1+a} < 1. \tag{1}$$

This nonideality, however, does not imply an irretrievable loss of information, because it turns out [2] to be possible to calculate from the experimental data of the joint measurement exactly *both* probability distributions of the interference and the path observables as these would be obtained in measurements having $a = 1$ and

$a = 0$, respectively. Evidently, by inserting the absorber in the interferometer the capacity of the experiment to yield information is increased, a measurement without absorber only yielding information on the interference observable. It was argued in [4] that, in order to exploit the additional information it is necessary to take into account the neutron detection rates in *both* outgoing beams. Because of (1), restriction to the information yielded by only one detector can easily give the false impression that the disturbance caused by the absorber must have deteriorated the information on the interference observable contained in the experiment. Although for both detectors the visibility of the interference pattern is decreased according to (1), nevertheless the *combined* quantum mechanical information of these patterns in the joint measurement is equivalent with the total information obtained if *ideal* measurements of interference ($a = 1$) and path ($a = 0$) are performed separately.

2 Absorber fluctuations

In [4] the influence of absorber fluctuations was considered, restricting to real fluctuations of a. It was demonstrated that, even though such fluctuations in general further reduce the visibility of the interference patterns (eqn. (18) of [4]), this does not imply that information on the interference observable is necessarily deteriorated by the fluctuations, because it is still possible to calculate the ideal information from the data of the joint measurement. A model of a fluctuating absorber due to Namiki and Pascazio [5] was used to illustrate this. This result is interesting because it illustrates the vast potential of the kind of measurements considered here to yield information on the incoming neutron state, notwithstanding fluctuations induced by the measurement arrangement have a decohering effect on the outgoing neutron state.

In a recent letter it was pointed out by Namiki and Pascazio [6] that a restriction to real fluctuations is not allowed. In the present note it will be demonstrated that our conclusions are not changed by an extension of the theory to complex fluctuations. We shall consider the experiment schematically represented in figure 1, in which χ is a phase shift induced from outside, and a an absorber. In order to take into account complex absorber fluctuations we must change the outgoing state (eqn.(9) of ref. [2]) according to

$$\begin{aligned} |\psi_{out}> &= \frac{1}{2}\{\alpha(-1 - Te^{i\chi}) + \beta(i - iTe^{i\chi})\}|\psi_1> + \\ &+\frac{1}{2}\{\alpha(-i + iTe^{i\chi}) + \beta(-1 - Te^{i\chi})\}|\psi_2> + \\ &+\frac{1}{\sqrt{2}}\sqrt{1 - |T|^2}(i\alpha - \beta)|\psi_3>, \\ |\alpha|^2 + |\beta|^2 &= 1, \end{aligned} \tag{2}$$

where $|\ \psi_1>, |\ \psi_2>$ and $|\ \psi_3>$ are three orthonormal neutron states, corresponding with, respectively, the neutron being transmitted towards detector D_1, towards detector D_2, and the neutron being absorbed. Writing

$$\bar{t} = \overline{|\ T^2\ |}, \tag{3}$$

$$\bar{T} = |\ \bar{T}\ |\ e^{i\phi} = \sqrt{\bar{t}(1-\epsilon)}e^{i\phi}, \tag{4}$$

where averaging is over the absorber fluctuations and ϵ is the decoherence parameter defined in [6], it is straightforward to calculated from (2) the detection probabilities of detectors D_1 and D_2 as

$$p_1 = |<\psi_1\ |\ \psi_{out}>|^2 = <\psi_{in}\ |\ \bar{\mathbf{M}}_1\ |\ \psi_{in}>, \tag{5}$$

$$p_2 = |<\psi_2\ |\ \psi_{out}>|^2 = <\psi_{in}\ |\ \bar{\mathbf{M}}_2\ |\ \psi_{in}>, \tag{6}$$

$\psi_{in} = \alpha\psi_2 + \beta\psi_2$ being the incoming state. We get:

$$\bar{\mathbf{M}}_1 = \frac{1}{2}[\mathbf{P}_+ + \bar{t}\mathbf{P}_- + \sqrt{\bar{t}(1-\epsilon)}\{\mathbf{Q}_A(\phi+\chi) - \mathbf{Q}_B(\phi+\chi)\}], \tag{7}$$

$$\bar{\mathbf{M}}_2 = \frac{1}{2}[\mathbf{P}_+ + \bar{t}\mathbf{P}_- - \sqrt{\bar{t}(1-\epsilon)}\{\mathbf{Q}_A(\phi+\chi) - \mathbf{Q}_B(\phi+\chi)\}]. \tag{8}$$

Here $\{\mathbf{P}_+, \mathbf{P}_-\}$ is the projection valued measure of the ideal path observable:

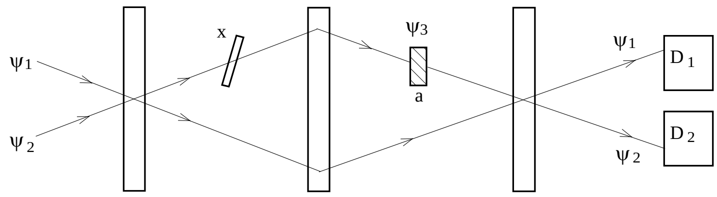

Figure 1: Three-slab neutron interference experiment with internal absorption.

$$\mathbf{P}_+ = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}, \ \mathbf{P}_- = \mathbf{I} - \mathbf{P}_+,$$

and $\{\mathbf{Q}_A(\phi+\chi), \mathbf{Q}_B(\phi+\chi)\}$ is the projection valued measure of the ideal interference observable with phase shift $\phi + \chi$:

$$\mathbf{Q}_A(\phi+\chi) = \begin{pmatrix} \sin^2 \frac{1}{2}(\phi+\chi) & \frac{1}{2}\sin(\phi+\chi) \\ \frac{1}{2}\sin(\phi+\chi) & \cos^2 \frac{1}{2}(\phi+\chi) \end{pmatrix}, \ \mathbf{Q}_B(\phi+\chi) = \mathbf{I} - \mathbf{Q}_A(\phi+\chi).$$

The POVM of the experimental arrangement can be found as the set of operators $\{\bar{\mathbf{M}}_1, \bar{\mathbf{M}}_2, \bar{\mathbf{M}}_3\}$ in which the expectation value of

$$\bar{\mathbf{M}}_3 = (1-\bar{t})\mathbf{P}_- \tag{9}$$

in the outgoing state $\mid \psi_{out} >$ is the absorption probability averaged over the fluctuations. If the incoming state is $\mid \psi_1 >$, then the detection probability p_1 of detector D_1 is given by

$$< \psi_1 \mid \bar{\mathbf{M}}_1 \mid \psi_1 >= \frac{1}{4}[1 + \bar{t} + 2\sqrt{\bar{t}(1-\epsilon)}\cos(\chi+\phi)], \tag{10}$$

which is the quantity customarily analyzed in neutron interference experiments (cf. eq. (19) of [6]).

It is clear from (7), (8) and (10) that the only consequence of *complex* fluctuations (compared to real ones) is an additional phase shift ϕ. Since this can easily be taken into account by means of a corresponding shift of the parameter χ, the case of complex fluctuations is not essentially different from the real case (note that, contrary to an assertion made in ref. [6], the possibility of loss of quantum coherence is *not* excluded ab initio for *real* fluctuations, because ϵ can be different from zero also in this case).

In order to demonstrate this similarity, we calculate the Wigner measure [2] for the general case of complex fluctuations. Whereas the POVM (7-9) describes information on the object that is distorted by the interaction with the measuring instrument, the Wigner measure describes ideal information. It can be obtained from the POVM by means of a deconvolution. In order to carry this out we define

$$\mathbf{R}_{11} = \mathbf{M}_1, \mathbf{R}_{12} = \mathbf{M}_2, \mathbf{R}_{21} = \mathbf{R}_{22} = \frac{1}{2}\mathbf{M}_3. \tag{11}$$

With (7),(8) and (9) it can straightforwardly be verified that

$$\sum_{\ell=1}^{2} \mathbf{R}_{k\ell} = \sum_{m=+,-} \bar{\lambda}_{km}\mathbf{P}_m, \tag{12}$$

$$\sum_{k=1}^{2} \mathbf{R}_{k\ell} = \sum_{n=A,B} \bar{\mu}_{\ell n}\mathbf{Q}_n(\chi+\phi), \tag{13}$$

with

$$(\bar{\lambda}_{km}) = \begin{pmatrix} 1 & \bar{t} \\ 0 & 1-\bar{t} \end{pmatrix},\ (\bar{\mu}_{\ell n}) = \frac{1}{2}\begin{pmatrix} 1+\sqrt{\bar{t}(1-\epsilon)} & 1-\sqrt{\bar{t}(1-\epsilon)} \\ 1-\sqrt{\bar{t}(1-\epsilon)} & 1+\sqrt{\bar{t}(1-\epsilon)} \end{pmatrix}. \tag{14}$$

The matrices (14) can be inverted to yield

$$\begin{aligned} (\bar{\lambda}^{-1}_{mk}) &= \frac{1}{1-\bar{t}}\begin{pmatrix} 1-\bar{t} & -\bar{t} \\ 0 & 1 \end{pmatrix}, \\ (\bar{\mu}^{-1}_{n\ell}) &= \frac{1}{2\sqrt{\bar{t}(1-\epsilon)}}\begin{pmatrix} 1+\sqrt{\bar{t}(1-\epsilon)} & -1+\sqrt{\bar{t}(1-\epsilon)} \\ -1+\sqrt{\bar{t}(1-\epsilon)} & 1+\sqrt{\bar{t}(1-\epsilon)} \end{pmatrix}. \end{aligned} \tag{15}$$

The Wigner measure is defined according to

$$\bar{\mathbf{W}}_{mn} = \sum_{k\ell=1}^{2} \bar{\lambda}^{-1}_{mk}\bar{\mu}^{-1}_{n\ell}\mathbf{R}_{k\ell}. \tag{16}$$

Like the Wigner distribution its expectation values have the properties of a quasi-probability distribution, the marginals of which are the probability distributions of the path and interference observables, i.e.,

$$\sum_{mn} \bar{\mathbf{W}}_{mn} = \mathbf{I}, \tag{17}$$

$$\sum_n \bar{\mathbf{W}}_{mn} = \mathbf{P}_m, m = +, -, \ \sum_m \bar{\mathbf{W}}_{mn} = \mathbf{Q}_n(\chi + \phi), n = A, B. \tag{18}$$

Hence, knowledge of the Wigner measure makes it possible to calculate the probability distributions of the ideal path and interference measurements. From (11), (15) and (16) we obtain

$$\begin{aligned}
\bar{\mathbf{W}}_{11} &= \frac{1}{2}[\mathbf{P}_+ + \mathbf{Q}_A(\chi + \phi) - \mathbf{Q}_B(\chi + \phi)], \\
\bar{\mathbf{W}}_{12} &= \frac{1}{2}[\mathbf{P}_+ - \mathbf{Q}_A(\chi + \phi) + \mathbf{Q}_B(\chi + \phi)], \\
\bar{\mathbf{W}}_{21} &= \bar{\mathbf{W}}_{22} = \frac{1}{2}\mathbf{P}_-.
\end{aligned} \tag{19}$$

This is, apart from the phase shift ϕ, identical with the Wigner measure obtained in [2] for the nonfluctuating absorber. Evidently, the absorber fluctuations do not prevent the experiment from yielding accurate quantum mechanical information on both the path observable and an interference observable that can be compared with the one measured in the nonfluctuating case by subtracting ϕ from the external phase shift χ.

From (16) it is clear that the Wigner measure only exists if both inverse matrices (15) exist. Hence, $\bar{t} = 1, \bar{t} = 0$, or $\epsilon = 1$ should be excluded. These are the parameter values for which one of the marginals (12) or (13) represents an *uninformative* observable [3]. In order to be able to calculate the Wigner measure (16) information on both the interference and the path observable is necessary. Hence the measurement arrangement must be carefully chosen so as to avoid the abovementioned parameter values.

3 Deterministic absorption

By Summhammer, Rauch and Tuppinger [1] the stochastic absorber discussed above is compared with a deterministic absorber consisting of a beam chopper which either leaves the particle undisturbed (probability v) or completely absorbs it (probability

$1 - v$). In [2] also this case was discussed. The POVM corresponding with this experiment is found according to

$$\begin{aligned}
\mathbf{M}'_1 &= \frac{1}{2}(1-v)\mathbf{P}_+ + v\mathbf{Q}_A(\chi), \\
\mathbf{M}'_2 &= \frac{1}{2}(1-v)\mathbf{P}_+ + v\mathbf{Q}_B(\chi), \\
\mathbf{M}'_3 &= (1-v)\mathbf{P}_-.
\end{aligned} \tag{20}$$

Defining, as before

$$\mathbf{R}'_{11} = \mathbf{M}'_1, \mathbf{R}'_{12} = \mathbf{M}'_2, \mathbf{R}'_{21} = \mathbf{R}'_{22} = \frac{1}{2}\mathbf{M}'_3, \tag{21}$$

an analogous analysis can be performed. We find (12) through (14) once again, with $\phi = 0$, and

$$\bar{t} = v, \epsilon = 1 - v, \tag{22}$$

implying

$$\overline{\mid T^2 \mid} = \mid \bar{T} \mid .$$

This latter result is to be expected, since in the deterministic absorber case T has value 1 or 0 with probability v or $1 - v$, respectively.

It is straightforward to calculate also the Wigner measure for this case. Once again we find (19) (with $\phi = 0$). Although the detailed interaction of object and measuring instrument are very different in the stochastic and the deterministic absorption cases, evidently the information that is left after the interactions have been taken into account by means of deconvolution, is essentially the same.

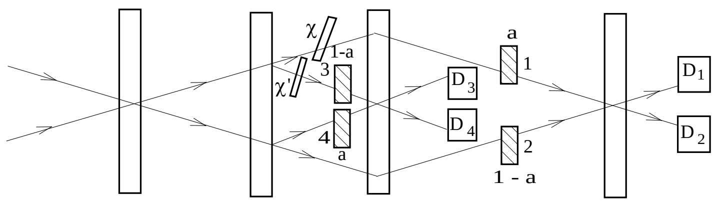

Figure 2: Complete neutron interference experiment.

4 A complete neutron interference measurement

All experiments discussed above are *trivial* joint measurements of path and interference because they do not yield any more information than is obtained in the separate measurements of the ideal path and interference observables. In particular, no correlations between values of the path and interference observables are obtained. In [7] a four slab neutron interference experiment was proposed that is nontrivial in this respect, and, hence, yields information on the correlation. In this experiment, which was constructed analogously to an optical experiment proposed by Busch [8], scatterers were used instead of an absorber. A four slab experiment using four absorbers as depicted in figure 2 may be easier to perform experimentally. It turns out to have the same informational properties. For nontriviality it is essential that there are two different phase shifts (χ and χ'). Detection rates of four detectors (D_1 through D_4) should be observed. Absorber fluctuations are left out of consideration. These could easily be introduced as before, without essentially influencing the information capacity of the experiment.

By determining, analogously to (2), the outgoing state $\mid \psi_{out} >$ we can calculate the detection probabilities $p_i =< \psi_{in} \mid \mathbf{M}_i \mid \psi_{in} >, i = 1,2,3,4$ of the four detectors, and the absorption probabilities $p_{Ai} =< \psi_{in} \mid \mathbf{M}_{Ai} \mid \psi_{in} >, i = 1,2,3,4$ in the four absorbers. We find

$$\begin{aligned}
\mathbf{M}_1 &= \frac{1}{4}[(1-a)\mathbf{P}_- + a\mathbf{P}_+ - \sqrt{a(1-a)}\{\mathbf{Q}_B(-\chi) - \mathbf{Q}_A(-\chi)\}], \\
\mathbf{M}_2 &= \frac{1}{4}[(1-a)\mathbf{P}_- + a\mathbf{P}_+ + \sqrt{a(1-a)}\{\mathbf{Q}_B(-\chi) - \mathbf{Q}_A(-\chi)\}], \\
\mathbf{M}_3 &= \frac{1}{4}[a\mathbf{P}_- + (1-a)\mathbf{P}_+ - \sqrt{a(1-a)}\{\mathbf{Q}_B(-\chi') - \mathbf{Q}_A(-\chi')\}], \\
\mathbf{M}_4 &= \frac{1}{4}[a\mathbf{P}_- + (1-a)\mathbf{P}_+ + \sqrt{a(1-a)}\{\mathbf{Q}_B(-\chi') - \mathbf{Q}_A(-\chi')\}], \qquad (23) \\
\mathbf{M}_{A1} &= \frac{1}{2}(1-a)\mathbf{P}_+, \\
\mathbf{M}_{A2} &= \frac{1}{2}a\mathbf{P}_-,
\end{aligned}$$

$$\mathbf{M}_{A3} = \frac{1}{2}a\mathbf{P}_+,$$
$$\mathbf{M}_{A4} = \frac{1}{2}(1-a)\mathbf{P}_-.$$

From (23) we find that

$$\sum_{i=1}^{4}\mathbf{M}_{Ai} = \frac{1}{2}\mathbf{I}. \tag{24}$$

Actually, transmission probabilities of the four absorbers were chosen such that (24) is fulfilled. This makes the total absorption rate independent of the incoming state, thus making the absorption rate irrelevant to a determination of this state by means of the experiment. Because of this the set of operators $\{\mathbf{R}_i = 2\mathbf{M}_i, i = 1,\dots,4\}$ constitutes a POVM fully describing the information obtained in the experiment.

By defining

$$\mathbf{R}_{11} = \mathbf{R}_1, \mathbf{R}_{12} = \mathbf{R}_2, \mathbf{R}_{21} = \mathbf{R}_3, \mathbf{R}_{22} = \mathbf{R}_4, \tag{25}$$

it can be shown that, analogously to (12) and (13), the marginals correspond with nonideal measurements of the path observable $\{\mathbf{P}_+, \mathbf{P}_-\}$ and the interference observable $\{\mathbf{Q}_A(-(\chi+\chi')/2), \mathbf{Q}_B(-(\chi+\chi')/2)\}$, with nonideality matrices

$$(\lambda_{km}) = \begin{pmatrix} a & 1-a \\ 1-a & a \end{pmatrix}$$

and

$$(\mu_{\ell n}) = \begin{pmatrix} \frac{1}{2} + \sqrt{a(1-a)}\cos(\frac{\chi-\chi'}{2}) & \frac{1}{2} - \sqrt{a(1-a)}\cos(\frac{\chi-\chi'}{2}) \\ \frac{1}{2} - \sqrt{a(1-a)}\cos(\frac{\chi-\chi'}{2}) & \frac{1}{2} + \sqrt{a(1-a)}\cos(\frac{\chi-\chi'}{2}) \end{pmatrix},$$

respectively. Inverting the matrices (λ_{km}) and $(\mu_{\ell n})$, the Wigner measure is found from (25), analogously to (16), as

$$\begin{aligned}
\mathbf{W}_{11} &= \frac{1}{2}\mathbf{P}_+ + \frac{\mathbf{Q}_A(-\chi') - \mathbf{Q}_B(-\chi')}{4(1-2a)\cos(\frac{\chi-\chi'}{2})} - \frac{a[\mathbf{Q}_A(-\frac{\chi+\chi'}{2}) - \mathbf{Q}_B(-\frac{\chi+\chi'}{2})]}{2(1-2a)},\\
\mathbf{W}_{12} &= \frac{1}{2}\mathbf{P}_+ - \frac{\mathbf{Q}_A(-\chi') - \mathbf{Q}_B(-\chi')}{4(1-2a)\cos(\frac{\chi-\chi'}{2})} + \frac{a[\mathbf{Q}_A(-\frac{\chi+\chi'}{2}) - \mathbf{Q}_B(-\frac{\chi+\chi'}{2})]}{2(1-2a)},\\
\mathbf{W}_{21} &= \frac{1}{2}\mathbf{P}_- + \frac{\mathbf{Q}_A(-\chi) - \mathbf{Q}_B(-\chi)}{4(1-2a)\cos(\frac{\chi-\chi'}{2})} - \frac{a[\mathbf{Q}_A(-\frac{\chi+\chi'}{2}) - \mathbf{Q}_B(-\frac{\chi+\chi'}{2})]}{2(1-2a)},\\
\mathbf{W}_{22} &= \frac{1}{2}\mathbf{P}_- - \frac{\mathbf{Q}_A(-\chi) - \mathbf{Q}_B(-\chi)}{4(1-2a)\cos(\frac{\chi-\chi'}{2})} + \frac{a[\mathbf{Q}_A(-\frac{\chi+\chi'}{2}) - \mathbf{Q}_B(-\frac{\chi+\chi'}{2})]}{2(1-2a)}.
\end{aligned} \tag{26}$$

It is straightforward to prove that the expectation values of these four operators in some state ψ completely determine this state.

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