The Copenhagen interpretation, and pragmatism[1]

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Abstract

In the past both instrumentalism and empiricism have inspired certain pragmatic elements into the Copenhagen interpretation of quantum mechanics. The relation of such pragmatisms with the correspondence principle is discussed. It is argued that neither Bohr nor Heisenberg did take 'correspondence' in one of these forms, and that it, in particular, was Bohr's classical attitude which caused him to apply in an inconsistent way his correspondence principle to the Einstein-Podolsky-Rosen experiment, thus causing much confusion. It is demonstrated that an empiricist pragmatism is conducive to an explanation of violation of the Bell inequalities as a consequence of 'complementarity' in the sense of 'mutual disturbance in a joint nonideal measurement of incompatible observables' rather than as being caused by 'nonlocal influences'.

1 Introduction

Let me first apologize for not being a philosopher, and, hence, not knowing precisely what in philosophic discourse is meant by pragmatism. For this reason in my discussion of the Copenhagen interpretation I will start from a physicist's notion of pragmatism in the sense of 'employing theory in a *practical* way so as to obtain useful results, without bothering too much about correspondence of theoretical entities with reality'. An *instrumentalist* interpretation of quantum mechanics, in which the mathematical formalism is considered to be merely an instrument for calculating probabilities of measurement results, seems to come close to this. As is well known, Bohr had an instrumentalist interpretation of the quantum mechanical wave function, to which he attributed a purely symbolic meaning only.

Instrumentalism is liable to criticism. Quantum mechanics is not pure mathematics. Like for every *physical* theory, some connection must be established with the physical reality it purports to describe. If this is omitted, doors are wide open for all kinds of confusions, as, indeed, have plagued the Copenhagen interpretation. Thus, an instrumentalist interpretation does not make a choice between the following two possible meanings of a 'quantum mechanical measurement result', mathematically

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represented by an eigenvalue a_m of an Hermitian operator A: i) a property of the microscopic object, ii) a pointer position of a measuring instrument. Defining an 'interpretation of a physical theory' as a 'mapping of the mathematical formalism of that theory into reality', I take it as one of my tasks to demonstrate that instrumentalism has caused failure of the Copenhagen interpretation to be a sound interpretation of quantum mechanics. As far as instrumentalism is pragmatic, this might be taken as a criticism of pragmatism.

In the literature also other reasons than instrumentalism can be found for attributing pragmatism to the Copenhagen interpretation. Thus, according to Stapp [1] the Copenhagen interpretation is pragmatic in the sense that quantum mechanical probabilities (or relative frequencies)

$$p_m = \langle\psi|E_m|\psi\rangle \text{ or } Tr\rho E_m \tag{1}$$

are not considered as referring to the *microscopic* object itself, but to *macroscopic* events obtained when a preparation procedure (represented by wave function ψ or density operator ρ) is followed by a measurement procedure (represented by the Hermitian operator $A = \sum_m a_m E_m,\ E_m$ projection operators). It, indeed, might be felt to be pragmatic to forgo all metaphysical discussion by being satisfied with describing 'just the phenomena' rather than 'microscopic reality itself' (e.g. by completely ignoring the relation between a click in a Geiger counter and the microscopic object causing that click to occur). In this sense correspondence does not seem to be inconsistent with pragmatism.

As we will see, at least Bohr's authoritative version of the Copenhagen interpretation is not in agreement with Stapp's characterization. Nevertheless, the latter is attractive due to its exclusive reliance on empirically accessible data, as well as because of its faithful representation of actual practice in experimental physics. For this reason it was recently proposed by the author [2] to amend the Copenhagen interpretation in such a way as to define a new interpretation, referred to as *neo*-Copenhagen interpretation, in which the empiricist kind of pragmatism attributed by Stapp to the Copenhagen interpretation is actually satisfied.

I prefer to refer to the characterization of the Copenhagen attitude proposed by Stapp as an '*empiricist interpretation* of the mathematical formalism of quantum mechanics', to be contrasted with a '*realist* interpretation' in which the mathematical entities of that formalism are thought to be mapped into *microscopic* reality. Thus,

Realist interpretation (fig. 1a):
Quantum mechanical observable A (in particular, its eigenvalues a_m), wave function ψ and density operator ρ refer to *properties* of the microscopic object.

Empiricist interpretation (fig. 1b):
Quantum mechanical observable A and its values a_m are *labels* of a *measurement*

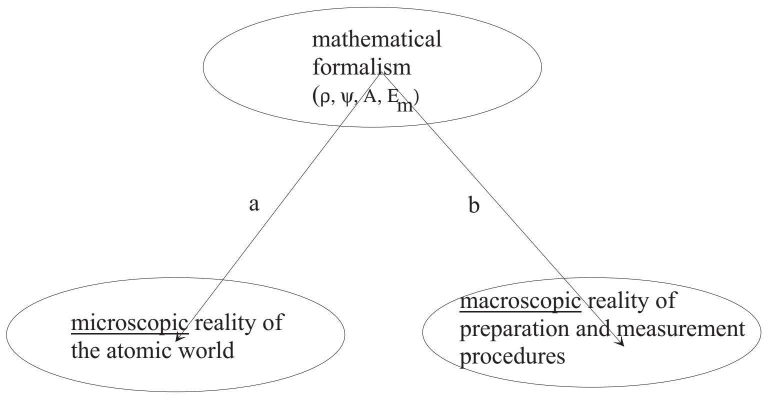

Figure 1: *Realist (a) and empiricist (b) interpretations of the mathematical formalism of quantum mechanics.*

procedure, and of the *pointer positions* of the measuring instrument, respectively; wave function ψ and density operator ρ are *labels* of *preparation procedures.*

An interpretation along Stapp's empiricist lines is very well possible (e.g. de Muynck [3]), and certainly has adherents among Copenhagen philosophers and physicists. However, neither Bohr nor Heisenberg –the founding fathers of the Copenhagen interpretation– were pragmatic in this empiricist sense. It is true that the measurement arrangement plays an important role in their quantum philosophies. But not in the empiricist sense given above. Notwithstanding their empiricist terminology in which is referred to a measurement as a 'quantum *phenomenon*', for both Bohr and Heisenberg a quantum mechanical observable refers to a property of the *microscopic* object, *not* to a pointer of a measuring instrument. As far as such instruments play a role in their reasonings, they serve to *define* (Bohr) or to *actualize* (Heisenberg) properties of the *microscopic* object.

In the following sections the three basic issues of the Copenhagen interpretation, viz. correspondence, completeness, and complementarity will be discussed from the point of view of an empiricist interpretation of the quantum mechanical formalism. This interpretation will also be applied to elucidate the problem of the Bell inequalities.

2 Correspondence

I will restrict myself to the mature form of Bohr's correspondence principle, developed after the mathematical formalism of quantum mechanics had largely been established (referred to in [3] as the *strong* form of correspondence, so as to distinguish it from the *weak* form requiring existence of a classical limit). This principle can be characterized by the following two requirements: i) experimental arrangement and measurement results have to be described in classical terms; ii) a quantum mechanical observable is exclusively defined within the context of the measurement serving to measure that observable.

Both points are liable to criticism. The first point has its origin in the logical positivist ideal of basing a theory on theory-independent observational data, so as to evade the vicious circularity caused by a dependence of the measurement on the very theory it is testing. For this reason, according to Bohr quantum mechanical measurement results should be expressed in terms of an independently tested theory, viz. classical mechanics. Nowadays we are convinced, however, that the requirement of theory-independence of observation statements cannot be met. Indeed, granting Bohr the necessity that a quantum mechanical measuring instrument have a *macroscopic* part, viz. a pointer, of which the position can be described by classical mechanics, we have become aware of the fact that it must also have a part which is sensitive to the *microscopic* information that has to be transmitted from the microscopic object to the measuring instrument (in order to be finally amplified to macroscopic dimensions). The microscopic process of information transfer (often called the pre-measurement) is actually the most important part of the measurement process; it should be described by quantum mechanics.

As to the second point the Copenhagen interpretation is liable to criticism because, notwithstanding the macroscopic part of the measuring instrument is playing an important role (as is seen from the first point), it completely ignores the measuring instrument *as a dynamically involved object*. In their discussions of the 'thought experiments' Bohr and Heisenberg are not so much interested in the final pointer position of the measuring instrument; their interest is rather directed toward properties of the *microscopic* object (discussed in classical terms) as these properties are *within the context of the measurement* (Bohr) or even as they are *after* the measurement has been completed (Heisenberg) (e.g. de Muynck [3], chapt. 4). Indeed, the Copenhagen interpretation of quantum mechanical observables is not an empiricist one as defined in sect. 1; on the contrary, the interpretation is a realist one, be it of a contextualistic kind: a quantum mechanical measurement result is thought to be a property of the microscopic object, be it *not* an *objective* property possessed already *before* the measurement (Einstein), but only well-defined *within the context of the measurement* (Bohr) or *after* the measurement (Heisenberg).

That the Copenhagen interpretation of Bohr and Heisenberg is not empiricist but contextualistic-realist can be concluded from many of their utterances; it can also most clearly be seen from Bohr's reaction [4] to the Einstein-Podolsky-Rosen (EPR) experiment [5], proposed to challenge the Copenhagen completeness thesis. This experiment was presented by EPR as a *measurement* of a property of particle 2 (cf. fig. 2a), *without letting this particle interact with a measuring instrument.* It is remarkable that it was accepted by Bohr as such a measurement, because the possibility of measuring a property of particle 2 by measuring a property of particle

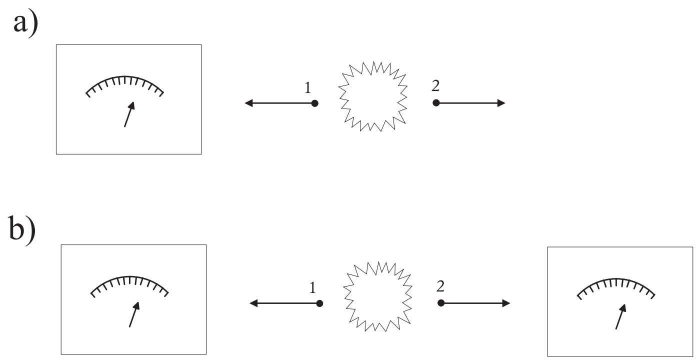

Figure 2: *a) EPR experiment; b) EPR-Bell experiment.*

1 must be based on the assumption that the correlation of these properties is well-defined. But, according to Bohr's correspondence principle such a correlation would be well-defined only within the context of a measurement of that correlation. However, a measurement of the correlation would require a measurement arrangement of the type depicted in fig. 2b, referred to as an EPR-Bell experiment because such measurements have been exploited to test the Bell inequalities (e.g. the experiments carried out by Aspect and coworkers [6])[2]. So, the fact that Bohr accepted the EPR experiment as a measurement of a property of particle 2 actually amounted to an inconsistent application by Bohr of his correspondence principle. Evidently, Bohr did not recognize 'correlation' as a quantum mechanical observable, to be well-defined only within the context of a correlation measurement. Most probably this oversight is a consequence of his *realist* interpretation of such physical quantities. Indeed, the Einstein-Bohr controversy, having its apotheosis in their discussion on the EPR paper [5], is about the question of whether such a *realist* interpretation of quantum mechanical observables can be *objectivistic* (Einstein: observables are thought to be independent of the 'observer *including his measuring instruments*'), or must be *contextualistic* (Bohr, Heisenberg: observables are thought to be dependent on the experimental arrangement).

In quantum mechanics textbooks 'measurement' is treated axiomatically; the measuring instrument is not dealt with in an explicit way. The quantum mechanical theory is treated as a description of *microscopic* reality. The wave function is thought not to refer to a preparation procedure, but to the *result* of such a procedure. By the same token a measurement result is not considered as referring to a property of a measuring instrument, but to a property of the microscopic object. Hence textbooks entertain a *realist* interpretation of the mathematical formalism of quantum mechanics, as do most quantum mechanical publications in scientific journals. Nowadays it is increasingly realized that such a realist interpretation cannot

[2]It should be noted that these experiments are often referred to as EPR experiments too, thus perpetuating the confusion originating with Einstein and Bohr.

be taken in Einstein's (or textbook) objectivistic sense, but should at least be contextualistic to be able to cope with no-go theorems like the Kochen-Specker theorem and the Bell inequalities.

Bohr's awareness of the important role of measurement in the interpretation of quantum mechanics certainly earns him the victory, widely attributed to him, over Einstein's objectivistic realism. However, we will see that a contextualistic realism alone is not able to solve all problems of interpretation: it will be necessary to accept the still weaker empiricist interpretation. As we shall see in sections 4 and 5, is the empiricist interpretation able to deal in a satisfactory way with Bohr's notion of complementarity (which is a key notion of quantum mechanics), as well as with the Bell inequalities. By relying on the empiricist interpretation it is not only possible to avoid the ambiguities stemming from an *instrumentalist* interpretation in which no choice is made between a (realist) property of the microscopic object and an (empiricist) pointer position of the measuring instrument (as is evidenced by the widely practiced confounding of EPR and EPR-Bell experiments), but it is also possible to evade dubious consequences of Bohr's contextualistic-realist interpretation.

Returning to the question of pragmatism, we can learn from this discussion of the Copenhagen correspondence principle that i) the Copenhagen interpretation is neither pragmatic in the instrumentalist sense (due to its reliance on 'correspondence'), ii) nor is it pragmatic in the empiricist sense (due to its '*realist* correspondence'). There undoubtedly is a certain pragmatic tendency in Bohr's contextualism, to the effect that one should be satisfied with 'knowledge about the microscopic object *as it is within the context of a measurement*' (since, due to the disturbing influence of measurement, *objective* knowledge in Einstein's sense is thought to be an unattainable ideal). However, this pragmatism should not be confused with the empiricist pragmatism defined above. Although Bohr's notion of 'quantum phenomenon' sounds deceivingly empiricist, it should in general not be equated with a 'measurement phenomenon' like a flash on a screen or a click in a counter. By sticking too much to the realist interpretation of classical mechanics (be it amended in a contextualistic sense) Bohr has not been able to benefit sufficiently from the possibilities the empiricist pragmatism has to offer. In particular, the empiricism of the neo-Copenhagen interpretation solves the problem that quantum mechanical measurement results did not exist before the measurement, but must come into being during that measurement: this is trivially satisfied if measurement results correspond to *final* pointer positions of measuring instruments.

3 Completeness

We have to distinguish two senses of completeness, viz. 'completeness in a wider sense' and 'completeness in a restricted sense', the first one turning around the question ‘Are hidden variables possible?’ whereas the second regards the question ‘Does quantum mechanics describe all measurements possible within the domain of atomic physics?’

3.1 Completeness in a wider sense

It is well-known that the EPR paper [5] was meant to challenge the Copenhagen completeness claim taken in a wider sense, by attempting to prove that quantum mechanical observables can play the roles of hidden variables, an objectively possessed value a_m being assumed to be simultaneously attributable to each observable in the initial state of the microscopic object (so-called ‘element of physical reality’). If possible this would support Einstein’s *ensemble* interpretation of the quantum mechanical wave function as against the Copenhagen *individual-particle* interpretation.

On the basis of the Kochen-Specker theorem [7], proven many years after the EPR paper was published, we can conclude that Einstein’s idea (that quantum mechanical measurement results a_m can be looked upon as objective properties of the microscopic object, possessed independently of the measurement, as is the case in classical mechanics) is not consistent with the mathematical formalism of quantum mechanics. At the time of the EPR discussion, however, Einstein could maintain his claim, and even strengthen it [8], by remarking that Bohr’s reproach [4] of ambiguity of the notion of ‘element of physical reality’ actually implied an unphysical consequence of ‘nonlocality’ which could be traded off against his conclusion of ‘incompleteness’: according to Einstein *locality* could be maintained if ‘*in*completeness’ of quantum mechanics is accepted; only if quantum mechanics were assumed to be complete, would nonlocality become an issue.

The festival of confusions involved in the EPR problem as discussed by Bohr and Einstein cannot be dealt with here in its entirety (see e.g. de Muynck [3], sect. 6.5). Suffice it to make two remarks. First, the possibility of an ‘ensemble interpretation’ is not at all thwarted by the failure of Einstein’s ‘elements of physical reality’ to be represented by quantum mechanical measurement results (e.g. Guy and Deltete [9]). There is no reason to exclude ensembles in which the EPR ‘elements of physical reality’ are represented by *sub*quantum properties (as is done in hidden-variables theories considered in later derivations of the Bell inequalities), rather than by quantum mechanical measurement results; particularly so if these latter refer to pointer positions of measuring instruments, as is the case in an *empiricist* interpretation. It seems that the possibility of subquantum theories, made respectable by John Bell’s opening towards experimental testing, has considerably changed physicist’s attitudes. Whereas a Copenhagen physicist A.D. 1935 would have answered the question of ‘completeness of quantum mechanics in the wider sense’ in the following vein: “Quantum mechanics is complete; there are no hidden variables”, would a neo-Copenhagen physicist A.D. 2007 give the converse answer that "Quantum mechanics is incomplete; hidden variables theories may be necessary to describe reality behind the phenomena." The latter would base his judgment on i) Bell's disproof [10] of the adequacy of von Neumann's 'no go' theorem ([11], sect. IV.2), ii) experimental evidence (e.g. [12]) provided by interference ('which way') experiments to the effect that an interference pattern, described by the wave function, is gradually built up out of local impacts of an *ensemble* of individual particles.

My second remark regards the nonlocality issue. For Bell the existence of Bohm's causal interpretation of quantum mechanics, considered to be a hidden-variables theory, and, hence, believed to be a palpable disproof of von Neumann's and other's 'no go' theorems, was a third reason for accepting the possibility of subquantum theories. Moreover, the nonlocality of Bohm's theory was reason for him to believe that the underlying reality described by such theories should exhibit nonlocal features, thus corroborating the nonlocality allegedly already present in the EPR experiment. Since it is questionable whether Bohm's interpretation of quantum mechanics does allow an interpretation as a hidden-variables theory (e.g. de Muynck [3], sect. 10.3), this third issue should, however, presumably be seen as a late act in the EPR festival of confusions.

It should finally be mentioned here that neither Bohr nor Heisenberg believed quantum mechanics to be 'complete in a wider sense'. We will return to the nonlocality issue in sect. 5, where it will become evident that, like the ensemble issue, also the nonlocality issue is a consequence of sticking too much to a *realist* interpretation of the notion of a quantum mechanical observable.

3.2 Completeness in a restricted sense

The answer to the question of 'whether quantum mechanics describes all measurements possible within the domain of atomic physics' is dependent on whether one restricts oneself to the *standard* formalism to be found in quantum mechanics textbooks, to the effect that quantum mechanical probabilities p_m satisfy the Born rule (1), in which the operators E_m are projection operators satisfying $E_m^2 = E_m$. If this *standard* quantum mechanics is meant, the answer is unambiguously "no". Indeed, experiments satisfying (1) do not exhaust all possible quantum mechanical measurements. On the contrary, most realistic measurements turn out to yield the more general probabilities

$$p_m = \langle\psi|M_m|\psi\rangle \text{ or } Tr\rho M_m,\ \sum_m M_m = I,\ M_m \geq O, \tag{2}$$

in which the operators M_m need not be projection operators. The set of operators $\{M_m\}$ defines a '*non*-orthogonal resolution of the identity', or a 'positive operator-

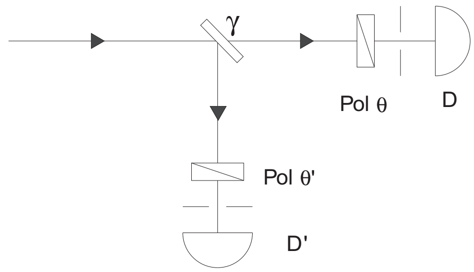

Figure 3: *Which-way polarization measurement of a photon.*

valued measure (POVM)'. Measurements satisfying (1) constitute a subset (corresponding to an 'orthogonal resolution of the identity', or 'projection-valued measure (PVM)') of the set of measurements satisfying (2). For instance, consider the 'which-way polarization measurement of a photon' depicted in fig. 3. When a photon impinges on a semitransparent mirror it has probability γ to be transmitted and probability $1-\gamma$ to be reflected. Hence, the detection probabilities of polarization detectors D and D′ are given by $P_{\rm D} = \gamma\langle\psi|E^{\theta}_{+}|\psi\rangle$ and $P_{\rm D'} = (1-\gamma)\langle\psi|E^{\theta'}_{+}|\psi\rangle$, respectively, in which E^{θ}_{+} and $E^{\theta'}_{+}$ are projection operators of the corresponding standard polarization observables. It is evident that the detection probabilities are not represented by these latter projection operators, but by the positive operators γE^{θ}_{+} and $(1-\gamma)E^{\theta'}_{+}$, respectively. Together with the operator $I-\gamma E^{\theta}_{+}-(1-\gamma)E^{\theta'}_{+}$ (representing the probability that a photon is absorbed in one of the analyzers), these operators define a POVM as employed in (2). Only in the exceptional cases in which γ is either 1 or 0 this POVM reduces to a PVM. Evidently, out of all possible experiments of this kind standard quantum mechanics merely covers a set of measure zero.

Taking into account only the experiments corresponding to the limiting values $\gamma = 1$ and $\gamma = 0$ (as is done in the Copenhagen interpretation as well as in most other interpretations of quantum mechanics) implies that *most* empirical information, to be obtained by means of measurements of the type depicted in fig. 3, is ignored. It seems to me that from a pragmatic point of view this must be utterly undesirable. Indeed, as will be illustrated in the following sections, reliance on too restricted a theory (viz. the *standard* formalism of quantum mechanics, describing only measurements corresponding to PVMs) has had a very large and probably rather dubious effect on the way physicists have been thinking about how reality is constituted. Although within the context of discovery it may have been advantageous to stick to the standard formalism of quantum mechanics in order to be able to obtain fast results by applying a relatively simple mathematical formalism, it seems that within the context of justification such pragmatism may have a restraining influence by causing stagnation due to misconceptions based on too scant empirical evidence.

4 Complementarity

Complementarity, as discussed within the Copenhagen interpretation, is about the (im)possibility of simultaneously measuring incompatible (standard) observables corresponding to noncommuting Hermitian operators. Allegedly, such measurements are impossible due to the disturbing influence of a measurement of observable A, say, on the measurement results to be obtained of the other observable B if $[A, B]_- \neq O$. By considering 'thought experiments' like double slit experiments and the γ-microscope, such a disturbance was unambiguously demonstrated to occur. This seemed to be corroborated by the possibility to derive from the mathematical formalism of quantum mechanics the well-known *Heisenberg uncertainty relation*

$$\Delta A \Delta B \geq \frac{1}{2} |\langle\psi|[A, B]_-|\psi\rangle|, \tag{3}$$

in which ΔA and ΔB are standard deviations in state ψ. For a long time the Heisenberg uncertainty relation (3) was considered to be an expression of the mutual disturbance of measurement results of *incompatible* observables A and B if these are measured simultaneously.

It lasted about 40 years before Ballentine [13] opposed this view by noting that the Heisenberg uncertainty relation (3) can be tested by *separate* measurements of observables A and B. Hence, this relation does not at all refer to a *simultaneous* measurement of A and B. Evidently, by equating the inequalities they had derived for a number of 'thought experiments' with the theoretical inequality (3) at hand, Bohr and Heisenberg had jumped to conclusions as regards the physical meaning of (3). Indeed, a reasonable interpretation of (3) could be that of a property of the *preparation procedure* of the *ensemble* represented by ψ (rather than a property of a measurement), more or less in Einstein's sense, to the effect that it is impossible to *prepare* an ensemble for which the physical quantities A and B are both dispersionless.

This does not imply, however, that the Bohr-Heisenberg idea of mutual disturbance in a simultaneous measurement of incompatible observables would be incorrect. Nowadays we are able to carry out realistic experiments, to be interpreted as joint (nonideal) measurements of incompatible standard observables, experimentally exhibiting such a disturbance (e.g. de Muynck [3], chapt. 8). But a quantum mechanical description of these experiments requires the generalization of the mathematical formalism of quantum mechanics referred to in sect. 3.2, encompassing measurements labelled by POVMs rather than by PVMs. In this section the measurement depicted in fig. 3 will be discussed as an example of such a joint nonideal measurement. By registering for each individual incoming photon the reactions of *both* detectors D and D′, the experiment gives occasion to define a *joint* detection probability $p_{mn},\ m, n = +$ or $-$, in which $p_{++} = 0$, $p_{+-} = p'_{\rm D}$, $p_{-+} = p_{\rm D'}$, and p_{--}

is the probability of a photon being absorbed in one of the analyzers. Writing

$$p_{mn} = \langle\psi|M^{\gamma}_{mn}|\psi\rangle \tag{4}$$

it follows that the POVM can be represented in the following bivariate form:

$$(M^{\gamma}_{mn}) = \begin{pmatrix} O & \gamma E^{\theta}_{+} \\ (1-\gamma)E^{\theta'}_{+} & 1-\gamma E^{\theta}_{+} - (1-\gamma)E^{\theta'}_{+} \end{pmatrix}. \tag{5}$$

By taking marginals it is possible to find the detection probabilities of each of detectors D and D$'$ separately, allowing to interpret the measurement as a joint nonideal measurement of the incompatible standard polarization observables in directions θ and θ', represented by the PVMs $\{E^{\theta}_{+}, E^{\theta}_{-}\}$ and $\{E^{\theta'}_{+}, E^{\theta'}_{-}\}$, respectively. We find, for incoming wave function ψ:

$$\begin{aligned} &\text{detector D}: \begin{pmatrix} \sum_n p_{+n} \\ \sum_n p_{-n} \end{pmatrix} = \begin{pmatrix} \gamma & 0 \\ 1-\gamma & 1 \end{pmatrix} \begin{pmatrix} \langle\psi|E^{\theta}_{+}|\psi\rangle \\ \langle\psi|E^{\theta}_{-}|\psi\rangle \end{pmatrix}, \\ &\text{detector D}': \begin{pmatrix} \sum_m p_{m+} \\ \sum_m p_{m-} \end{pmatrix} = \begin{pmatrix} 1-\gamma & 0 \\ \gamma & 1 \end{pmatrix} \begin{pmatrix} \langle\psi|E^{\theta'}_{+}|\psi\rangle \\ \langle\psi|E^{\theta'}_{-}|\psi\rangle \end{pmatrix}. \end{aligned} \tag{6}$$

In these expressions the nonideality matrices $\begin{pmatrix} \gamma & 0 \\ 1-\gamma & 1 \end{pmatrix}$ and $\begin{pmatrix} 1-\gamma & 0 \\ \gamma & 1 \end{pmatrix}$ represent the nonidealities in the determination of the probabilities of the standard polarization observables in directions θ and θ', respectively, yielded by the present experiment.

It is important to notice the complementary behaviour of the two nonideality matrices given above: the quality of the information yielded by one marginal probability distribution increases as that of the other marginal probability distribution decreases by changing the value of γ. Indeed, for $\gamma = 1$ information on the standard polarization observable in direction θ is ideal whereas that on the standard polarization observable in direction θ' is maximally nonideal. For $\gamma = 0$ the opposite holds. For $0 < \gamma < 1$ information on both standard observables is nonideal to a certain extent. Denoting the two nonideality matrices that are involved by $(\lambda_{mm'})$ and $(\mu_{nn'})$, respectively, and taking the average row entropy

$$J_{(\lambda)} = -\frac{1}{N}\sum_{mm'} \lambda_{mm'} \ln \frac{\lambda_{mm'}}{\sum_{m'} \lambda_{mm'}}$$

as a measure of the nonideality expressed by the matrix $(\lambda_{mm'})$ (and analogously for $(\mu_{nn'})$), by Martens [14] an inequality was derived, for the present measurement reading

$$J_{(\lambda)} + J_{(\mu)} \geq -\ln\{\max_{mn} Tr E^{\theta}_{m} E^{\theta'}_{n}\}. \tag{7}$$

In fig. 4 the curved line is a parametric plot of $J_{(\lambda)}$ versus $J_{(\mu)}$ as a function of γ. The shaded area contains the values of $J_{(\lambda)}$ and $J_{(\mu)}$ forbidden by the Martens inequality (7).

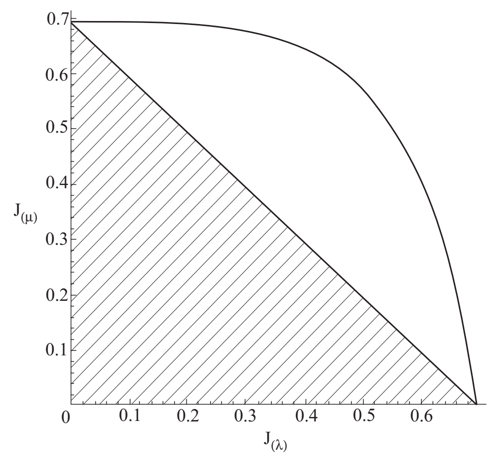

Figure 4: *Parametric plot of* $J_{(\lambda)}$ *versus* $J_{(\mu)}$ *as a function of* γ.

Contrary to the Heisenberg inequality (3) the Martens inequality (7) yields a faithful representation of the idea of mutual disturbance in a simultaneous measurement of incompatible standard observables, as found in the early discussions on the 'thought experiments'. In particular, it can be seen that the Martens inequality, being completely independent of the initial state vector or density operator, refers to the *measurement procedure* alone, and can be understood as a consequence of the mutual exclusiveness of measurement arrangements for measuring incompatible standard observables. Hence, the Martens inequality does execute the function erroneously attributed to the Heisenberg inequality. Evidently, the physical intuitions of Bohr and Heisenberg were sufficiently adequate. However, they lacked the mathematical formalism (i.c. the formalism of positive operator-valued measures) necessary for a mathematical expression of these intuitions. Due to the restriction of the formalism to standard observables, and the corresponding restriction of the domain of application of quantum mechanics, they were not able to see that the notion of complementarity actually consists of two different forms, viz. one for *preparation* and one for *measurement*, the former being represented by the Heisenberg uncertainty relation, the latter by the Martens inequality.

This clearly illustrates the risk, already pointed at in sect. 3.2, that a too narrow scope of the domain of interest may lead to confusion. Moreover, the example of the POVM (5) demonstrates that a restriction of the empiricist interpretation to the *standard* formalism may generate the erroneous idea that an ideal measurement of a standard observable does not yield any information on an incompatible one. As a matter of fact, it is easy to verify that from (6) we find for $\gamma = 1$ (i.e. for the ideal measurement of the standard polarization observable in direction θ) that $\sum_m p_{m-} = 1$. This implies that the measurement result '$-$' can be attributed *with certainty* to the standard polarization observable in direction θ'. Evidently the maxim 'Unperformed experiments have no results' (e.g. Peres [15]), allegedly making it possible to deny the existence of the value of the polarization observable in direction θ' if the polarization observable in direction θ is actually measured, is too shallow. The simultaneous existence of measurement results for the standard polarization observables in directions θ and θ' will be used in the next section to analyze the problem of the Bell inequalities.

5 Bell inequalities

As mentioned in sect. 3.1 the EPR experiment has induced the idea that the reality described by quantum mechanics should have some feature of nonlocality. On the basis of this assumption inequalities were derived by Bell [16], allegedly to be violated only in case of nonlocality. Bell's expectation is still widely thought to be corroborated by the EPR-Bell experiments performed by Aspect and coworkers [6], in which a violation of the Bell inequalities was experimentally found. In this section it is demonstrated that this conclusion, too, is based on the restricted view of quantum mechanics discussed in sect. 3.2, and, hence, too shallow.

Let us first see why it is rather improbable that there is any causal relation between violation of the Bell inequalities and nonlocality. As a matter of fact, violation of the Bell inequalities is *only* possible if *not* all of the four standard observables A_1, B_1, A_2 and B_2 that are involved, are mutually compatible (since the existence of a quadrivariate probability distribution, as a consequence of compatibility, would imply satisfaction of the Bell inequalities). Hence, violation of the Bell inequalities is a consequence of *in*compatibility of some of the observables that are involved. But, as a consequence of the principle of local commutativity (prescribing commutativity of observables measured in causally disjoint regions of space-time) is *in*compatibility a *local* affair. Only measurements performed in *one and the same region* of space-time can be incompatible, and, hence, disturb each other so as to cause violation of the Bell inequalities. This well-known result is often ignored on the basis that the Bell theorem is not a theorem of quantum mechanics but a hidden-variables theorem. However, it would be rather far-fetched to believe that two different theories, valid in the same experimental domain, would yield diametrically opposed explanations of the same phenomena. As will be seen in the following, the explanation of violation of the Bell inequalities on the basis of *local* disturbances will be corroborated by the generalization of the mathematical formalism introduced in sect. 3.2.

Let us consider the *generalized* Aspect experiment depicted in fig. 5. The Aspect experiments [6] are special cases of this experiment, in which $(\gamma_1, \gamma_2) = (1,1), (1,0), (0,1)$ or $(0,0)$. Comparing the experiment with the one depicted in fig. 3, and taking into account our discussion in sect. 4, we see that the generalized Aspect experiment can be interpreted as a joint nonideal measurement of four

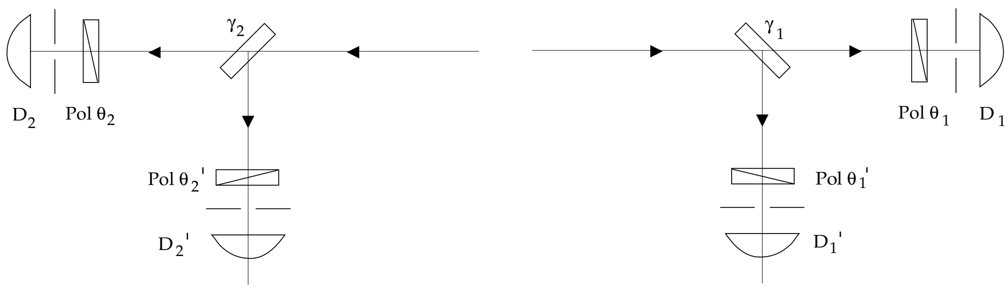

Figure 5: *Generalized EPR-Bell experiment.*

standard polarization observables. The quadrivariate probability distribution of the four detectors D_1, D'_1, D_2 and D'_2 is found completely analogously to (4) as the expectation value of a quadrivariate POVM:

$$p^{\gamma_1\gamma_2}_{m_1n_1m_2n_2} = \langle\psi|M^{\gamma_1\gamma_2}_{m_1n_1m_2n_2}|\psi\rangle. \tag{8}$$

It is also not difficult to see that this POVM is just the direct product of the bivariate POVMs (5) of the joint nonideal measurements performed in each of the arms of the interferometer:

$$M^{\gamma_1\gamma_2}_{m_1n_1m_2n_2} = M^{\gamma_1}_{m_1n_1}M^{\gamma_2}_{m_2n_2}. \tag{9}$$

It is important to note that, due to the existence of the quadrivariate probability distribution (8), its four bivariate marginals describing correlations between photons 1 and 2 do satisfy the Bell inequalities. This, actually, is a simple consequence of the fact that each measurement on an individual photon pair yields a *quadruple* of measurement results, one for each of the four detectors. Note that this holds true also for each of the special values of the parameters γ_1 and γ_2 employed in the Aspect experiments [6]. Indeed, violation of the Bell inequalities in these latter experiments is a consequence of the fact that for the measurement results of these experiments (constituting octuples rather than quadruples) no quadruples can be found.

It is often thought that the explanation of the nonexistence of such quadruples and, hence, of the ensuing violation of the Bell inequalities in the Aspect experiments, is provided by a nonlocal influence of the measurement arrangement for photon 1 on the measurement results for photon 2 (and vice versa). As already put forward, this explanation is not very plausible. There is a much more plausible explanation, though, viz. the mutual disturbances of measurement results in each of the arms of the interferometer *separately*, as a consequence of the fact that, even for $\gamma_i = 1$ or 0, the measurements can be interpreted as joint nonideal measurements of incompatible observables, which are mutually disturbing. Thus, it is evident that the measurement result of the standard polarization observable of photon 1 in direction θ_1 in case $\gamma_1 = 1$ will in general be different from the value found if $\gamma_1 = 0$, even if the individual preparations are identical. Hence, the nonexistence of one single quadruple of measurement results for the four measurements realized in an Aspect experiment need not be a consequence of nonlocal influences, but can be attributed to changing measurement results of detectors D_1 and D'_1 if γ_1 is switched from 1 to 0 (and analogously for D_2 and D'_2 if γ_2 is switched). Hence, disturbances in one arm of the interferometer are caused by changing the measurement arrangement in that same arm. Once again it is seen that a natural explanation of a phenomenon may be overlooked by sticking to a too restrictive experimental and theoretical domain: in the standard formalism it is not at all obvious that violation of the Bell inequalities can be seen as a consequence of complementarity rather than nonlocality.

6 Conclusions

In a physicist's approach to pragmatism, instrumentalist and empiricist interpretations of the mathematical formalism of quantum mechanics have been set against the widely used realist interpretation. It is concluded that, in developing the Copenhagen interpretation, both Bohr and Heisenberg entertained such a realist interpretation, be it of a contextualistic kind (as opposed to Einstein's objectivistic realism). It is argued in sec. 2 that by this realism Bohr was seduced into *inconsistently* applying his correspondence principle to the EPR problem, thus causing much confusion. It is demonstrated that drawing a clear distinction between the 'EPR experiment' and 'EPR-Bell experiments devised for testing the Bell inequalities' is crucial to lifting the above-mentioned confusions, in particular the nonlocality conundrum. Thus, in the empiricist version of pragmatism a consistent application of Bohr's correspondence principle would not have given rise to any idea of nonlocal influences.

In sec. 3 two senses of completeness of quantum mechanics are distinguished. First, 'completeness in a wider sense', denying the possibility of hidden-variables theories, attributed to the Copenhagen interpretation as one of its main features. In sec. 3.1 it is argued that, contrary to widespread belief, it is very well possible that in a pragmatic approach the Copenhagen 'completeness thesis in a wider sense' be replaced by an *ensemble* interpretation of the wave function. This, actually, is the default way nowadays quantum mechanical experiments are dealt with in experimental practice. Here pragmatism seems to side with scientific progress by cutting off deliberations regarding currently impracticable tests of subquantum theories.

With respect to 'completeness in a restricted sense' the situation is different. The question is here whether the domain of application of quantum mechanics is restricted to measurements described by the *standard* formalism. As discussed in sections 3.2, 4 and 5, the answer to this question is negative since experiments can easily be performed needing a *generalization* of the mathematical formalism for their description. It, however, seems that a certain pragmatism induces physicists to stick to the better-known standard formalism, even though considerably more insight can be obtained from the more general experiments described by the generalized formalism. This is illustrated in sections 4 and 5 by a discussion of the Copenhagen notion of ‘complementarity’ and by applying a *generalized* EPR-Bell experiment to the problem of the Bell inequalities. It is seen that only the *generalized* formalism mathematically encompasses the notion of ‘mutual disturbance in a joint nonideal measurement of incompatible observables’, and that violation of the Bell inequalities is a consequence of this same *local* incompatibility rather than being caused by nonlocal influences.

References

[1] H.P. Stapp, *Am. J. Phys.* **40**, 1098 (1972).

[2] W.M. de Muynck, *Found. of Phys.* **34**, 717-770 (2004).

[3] Willem M. de Muynck, *Foundations of quantum mechanics, an empiricist approach*, Fundamental theories of physics, vol. 127, Kluwer Academic Publishers, Dordrecht, Boston, London, 2002.

[4] N. Bohr, *Phys. Rev.* **48**, 696 (1935).

[5] A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).

[6] A. Aspect, P. Grangier, and G. Roger, *Phys. Rev. Lett* **47**, 460 (1981); A. Aspect, J. Dalibard, and G. Roger, *Phys. Rev. Lett.* **49**, 1804 (1982).

[7] S. Kochen and E.P. Specker, *J. Math. and Mech.* **17**, 59 (1967).

[8] A. Einstein, *Dialectica* **2**, 320 (1948).

[9] R. Guy and R. Deltete, *Found. of Phys.* **20**, 943 (1990).

[10] J.S. Bell, *Rev. Mod. Phys.* **38**, 447 (1966).

[11] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Springer, Berlin, 1932.

[12] G. Möllenstedt, C. Jönsson, *Zeitschr. f. Phys.* **155**, 472 (1959).

[13] L.E. Ballentine, *Rev. Mod. Phys.* **42**, 358 (1970).

[14] H. Martens and W. de Muynck, *Found. of Phys.* **20**, 255, 357 (1990).

[15] A. Peres, *Am. J. Phys.* **46**, 745 (1978).

[16] J.S. Bell, *Physics* **1**, 195 (1964).