ON MEASUREMENT IN RELATIVITY THEORY[1]

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1. INTRODUCTION

The present paper, which treats the interpretation of measurement in relativity theory (RT), is primarily inspired by the author's experience in quantum mechanics (QM) cf., [1, 2, 3]. In this latter theory the problem of measurement is as old as the theory itself, the role played by the measuring instrument in its interpretation being emphasized by Bohr.

A physical theory consists of a mathematical formalism together with an interpretation. An interpretation of the theory is a mapping of mathematical quantities into physical reality. In QM there are two possibilities for such a mapping, corresponding to two different interpretations, viz., the *realist* interpretation, mapping the theory into *microscopic* reality and considering it as a description of properties of the microscopic entities, and the *empiricist* interpretation in which the quantities are mapped into the *macroscopic* reality of what is often indicated as the *phenomena*. Whereas in the former interpretation quantum mechanical observables and state functions are viewed as describing properties of the microscopic objects themselves, in the latter interpretation these concepts are just labels of certain measurement and preparation procedures within the domain of application of the theory. In the empiricist interpretation the theory is thought to describe merely (cor)relations between certain (macroscopic) preparations and pointer positions of (equally macroscopic) pointers of measuring instruments, leaving the description of the microscopic world to subquantum theories to be developed later. It is important to note that the phenomena (like tracks in a Wilson cloud chamber, or light flashes in a photon counter), although being induced by microscopic entities, are to be considered as properties of the macroscopic instruments rather than as properties of the microscopic objects. In an empiricist interpretation of quantum mechanics the quantum mechanical formalism is considered as a 'surface' or 'experimental' model as defined by van Fraassen ([4], p. 113), the density operator ρ representing the condition of preparation and the observable (positive operator-valued measure) $\{M_m\}$ corresponding to the condition of measurement. The probability distribution $\{Tr\rho M_m\}$ corresponds to van Fraassen's surface state representing just the experimental data registered by the measuring instrument.

It is clear that the custom of referring to a quantum mechanical quantity as an 'observable' has an empiricist origin. Indeed, in developing matrix mechanics Heisenberg emphasized the empiricist meaning of the formalism, denying classical notions like particle trajectories any right of existence. Under the influence of logical empiricism this understanding of QM has been very popular among physicists.

[1]Revised version of a paper published in *Recent advances in relativity theory*, Vol. **2**, M.C. Duffy and M. Wegener, eds., Hadronic Press, Florida, 2001, pp.156-168.

Yet, the empiricist understanding has also been a matter of concern. Indeed, for most physicists the realist interpretation of quantum mechanics is the more plausible one. They consider it self-evident that QM, being devised especially to describe phenomena related to the existence of microscopic particles, must describe just these particles, much in the same way as classical mechanics is thought to describe macroscopic objects. This latter analogy has given rise to the idea that the quantum mechanical wave function should replace the classical phase space point as a description of a microscopic particle, and, hence, is related to the particle itself rather than to macroscopic preparation phenomena. Analogously, the quantum mechanical observable is viewed upon as a property of the microscopic particle rather than as a (directly observable) property of the measuring instrument. This tendency towards a realist interpretation is often combined with an empiricist locution, causing quite a bit of confusion. Thus, a transition from one stationary state of an atom to another one is often referred to as a phenomenon, as if it would be possible to observe directly the atom transiting from one stationary state to the other. It is seldom recognized that what is actually observed is not even the photon accompanying the transition, but some event triggered by the photon in a detector (like e.g., an electric pulse in a proportional counter). Often in the quantum mechanical literature it is rather lip service payed to empiricism when empiricism is professed at all. A notable exception to this tendency is Wheeler who's "No elementary phenomenon is a phenomenon until it is an observed phenomenon"[5] seems to be an empiricist credo for QM.

Nowadays, however, there is a tendency even to abolish any appearance of empiricism, and to entertain a downright realist interpretation. Quantum mechanical particles are viewed as wave packets flying around in space. By the same token particles are thought to possess as a property the value of momentum that is found on measurement, even though it is well-known that the attribution, independently of measurement, of well-defined values to all quantum mechanical observables would bring about a discrepancy since it would cause the Bell inequalities to be satisfied [2]. A reason for this tendency towards a realist interpretation of the formalism might be that we do require from our physical theories not only a faithful description of the phenomena but also *explanations*. Thus, Peres [6] seems to equate an empiricist interpretation with obedience to the maxim "Thou shalt not think", because this interpretation is leaving the reality behind the phenomena completely out of consideration. Those who cannot refrain from thinking will, according to Peres, look for causal explanations on the basis of microscopic processes.

Although I strongly endorse Peres's plea for explanations, I nevertheless prefer an empiricist interpretation of QM. There are several reasons for this. First, it should be noted that not equating electron and wave packet does *not* imply the non-existence of electrons (as would be required e.g., by an idealist philosophy). Questions regarding the existence of the moon when nobody looks, once jokingly put by Einstein, may have been hotly debated in earlier centuries, but are no longer interesting in 20$^{\text{th}}$ century physical discourse. Physicists believe that electrons exist independently of any observation, and bring about causal relations between preparation and measurement in physical experiments. The question only is whether this independent existence is described by quantum mechanics, or whether we will need to devise new theories for such a description. It should be noted that Einstein has contributed considerably to the acceptance of a realist interpretation of QM by his resistance against the Copenhagen completeness claim, thus promoting the *individual-particle* interpretation of the wave function to be replaced by a (tentatively realist) *ensem-*

ble one. Contrary to Einstein's expectation, however, this replacement does not completely solve all interpretational problems of quantum mechanics.

A second argument against a realist interpretation of QM is its negligence of the role played by the measuring instrument. It should be stressed that decisive for the question whether theory and experiment are in agreement is whether the relative frequencies, calculated from the mathematical formalism of quantum mechanics, correspond to the (relative) number of times the pointer of a measuring instrument points at a certain position. Whether a pointer position corresponds to a property of the microscopic object is a matter of interpretation. In particular, if the object is not thought to have any property prior to measurement (as is thought in the Copenhagen interpretation), then a realist interpretation has a problem (sometimes tentatively solved by attributing the value of the measured observable to the object either *in the context of the measurement* (contextualistic realism) or even as a property possessed by the object *after the measurement*). In any case, in a realist interpretation the observable has a double role since its empirical relation to the measuring instrument must remain valid next to its interpretation as a microscopic property. From a methodological point of view such a double role is dubious.

Finally, there is the practical problem that the picture of reality conveyed to us by the measurement may be not a completely faithful one. For a realist interpretation of a physical theory it is necessary that we can attribute the quantities of our theory to the object itself as properties, thus assuming that no deformation of the information about the object is introduced by the observation. In an empiricist interpretation the possibility is left open that a theory may yield only a distorted description of reality. In an empiricist interpretation the theory only describes the *output* of measurement procedures. Whether this output is a faithful representation of reality (the input) need not be left as an open question, but may be submitted to a more detailed investigation of the process of observation. In QM this has led to a generalization of the theory, in which quantum mechanical observables are represented by positive operator-valued measures rather than by Hermitean operators, and to the development of a theory of non-ideal measurements [1] providing considerably more insight into quantum mechanical complementarity than was possible on the basis of the usual axiomatics based on observables represented by Hermitean operators.

2. EMPIRICIST AND REALIST INTERPRETATIONS OF RT

In Einstein's foundational paper [7] on RT strong empiricist influences can be observed. Thus, according to Holton ([8], p. 224) in this article Einstein is identifying reality with what is given by sensations, the 'events', rather than putting reality on a plane beyond or behind sense experience. Einstein's empiricist attitude is illustrated by Holton by two quotations from Einstein's 1905 paper : "The 'time' of an event is that which is given simultaneously with the event by a stationary clock located at the place of the event", and, "We have to take into account that all our judgements in which time plays a part are always judgements of simultaneous events. If for instance I say, 'that train arrived here at seven o'clock', I mean something like this: 'the pointing of the small hand of my watch to seven and the arrival of the train are simultaneous events'".

Remarks of a comparable empiricist nature can be found in Einstein's writings over the years. Thus, still in his 1916 synopsis of the foundation of the General Theory of Relativity [9] it is stated that "All our space-time verifications invariably amount to a determination of space-time coincidences". According to Friedman ([10], p. 29 f.f.) this passage even contains the beginnings of the empiricist and verificationist interpretations of science characteristic of later positivism. A further testimony of the empiricist flavour of RT is given by Frank ([11], p. 269): "The definition of simultaneity in the special theory of relativity is based on Mach's requirement that every statement in physics has to state relations between observable quantities". It is also well-known that Einstein's expulsion of the aether from physical theory is often hailed as an 'antimetaphysical' act.

A consistent development into the empiricist direction would have yielded a picture in which RT is thought to describe only readings of clocks and measuring rods and relations between such readings. In this view the metric tensor does not describe the structure of space-time, but it is merely a summary of the behaviour of measuring rods and clocks in different parts of space-time. The possibility of such a view was demonstrated by Marzke and Wheeler [12]. In the empiricist view RT is about measurements of time and distance, defined operationally. Thus,

time = 'the position of the small hand of my watch';
length = position difference between two markers on a (rigid) measuring rod superposed on the object.

However, already in Einstein's 1905 article we find premonitions of a different trend in which sensory experiences are not the final building blocks of reality (cf. Holton [8], p. 225). For instance, the postulate of the constancy of the velocity of light can hardly be considered to be derived from sensory experience. Especially after his 1913 paper with Grossmann [13] Einstein seems to endorse the view that "it could no longer be required that coordinate differences (ds) should signify direct results of measurements with ideal scales or clocks" ([14], p. 288).

A fundamental change in Einstein's way of thinking about RT was observed by Bridgman ([15], p. 335): "...Einstein did not carry over into his general RT the lesson and insights which he himself has taught us in his special theory". It is above all the development of the four-dimensional space-time formulation of STR and GTR that convinced Einstein that the space-time metric describes objective reality, not just Mach's sensory experience. The divergence between Mach and Einstein with respect to RT is well-known (Holton [8], Frank [11]). It seems that at this moment the prevailing attitude towards RT, like in quantum mechanics, is a realist one. This even seems to hold true with Wheeler, whose attitude, as far as QM is not involved, with respect to space-time seems to be a realist one ("space-time is a purely classical concept. It is a classical history of space geometry changing with the progress of time" [16]), notwithstanding the Marzke-Wheeler operationalization. The space-time manifold (or the spatial manifold) is considered as a model of an objectively existing space-time (or non-Euclidean three-space). Geodesic motion,

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = 0,$$

is considered as an objective description of the trajectory of a particle through space-time, in which the coordinates x^1, x^2, x^3 just indicate the spatial position of the particle expressed in some coordinate frame. Of course it is conceded that the coordinates themselves do not have physical significance, and that it does not make sense to ask for the 'real' length of an object. However, those quantities that are the same for every observer in any coordinate frame, the invariants like $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$, are generally considered as properties of the object, *not* as mere differences between markers on a measuring instrument (as would suit an empiricist interpretation).

Presumably this kind of realism may have been in Einstein's mind when he compared in later years the space-time metric to a new kind of aether, deviations from geodesic motion being comparable to absolute acceleration with respect to Newton's absolute space. John Bell ([17], chapt. 9) seems to go even one step further by suggesting that the Lorentz contraction is a real consequence of the deformation of the electromagnetic field of a moving charged particle as compared to a static one, the only reason for its unobservability being that all objects, including measuring rods contract in the same way.

Summarizing, also in RT a tendency can be observed from an empiricist interpretation to a more realist one, in which the role of the observer is minimized as much as possible. Tensors nowadays are geometrical objects, with an emphasis on the word 'objects', and only tensors are thought to have physical relevance. Thus, the metric tensor is thought to describe the structure of the four-dimensional 'container' or embedding space with respect to which absolute motion (that is, absolute acceleration and rotation) can be retained in RT (Friedman, [10], p. 63 and p. 232). The relation to *measurement* of time and length is seldom discussed in modern texts on RT. Admittedly, the terminology used is often vaguely empiricist, a description of some process in terms of a specific coordinatization being interpreted as 'a description of the process as *seen* by an observer in the specified coordinate frame'. However, generally it is not analyzed what is meant by the observer's 'seeing' . What is 'seen' is attributed as a property to the object. If the object is 'seen' at the space-time point (x^0, x^1, x^2, x^3) this is interpreted as the object *being* at this point. It is seldom realized that 'seeing' is a physical process of image formation in which the image is fundamentally different from the object.

One notable exception is Bridgman in the closing sentence of his contribution to the Einstein celebration volume [15]: "That in his conviction of the possibility of getting away from any special coordinate system, in his conviction of the fruitfulness of so doing, and in his treatment of the event as something primitive and unanalyzed, he (i.e., Einstein) has carried into general RT precisely that uncritical, pre-Einsteinian point of view which he has so convincingly shown us, in his special theory, conceals the possibility of disaster". Bridgman evidently is worried about Einstein's preoccupation with physics as a description of an *objective* reality, i.e., a description in which the process of observation is completely left out of consideration. Presumably Bridgman was aware of the danger that this neglect of the observation process could easily shift the meaning of a tensor component from 'a property as *seen* by an observer in his coordinate frame', to 'a property the object *has* relative to the observer in his coordinate frame'. Such a shift would imply a fundamental change in the philosophical significance of the theory. The former view could be reconciled with the existence of an objective, observer-independent reality, the process of observation being responsible for the different results obtained by observers in different reference frames (e.g., length measurements by observers having different relative velocities). On the latter view an objective reality would be impossible. Every observer would have his own *relative* reality. According to Whitehead [18] it would amount to a 'fallacy of misplaced concreteness' to suppose the existence of one single objective entity behind this multitude of relative reali-

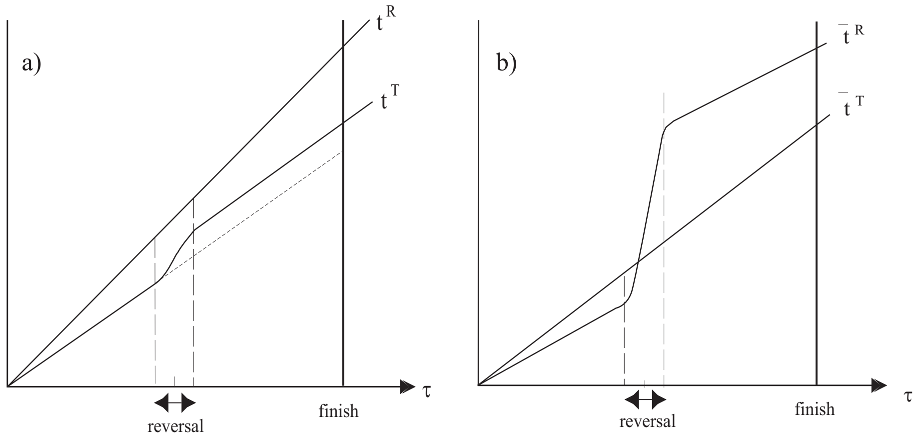

Figure 1: a): Readings of T's and R's clocks: by R (a); by T (b).

ties; the event is just the complex of all aspects represented in the different relative realities (Whitehead's theory of multiple space-time systems).

As Whitehead [18] remarks, Einstein would presumably reject his theory of multiple space-time systems, since for him *objective* reality was sacred. It seems to me that Einstein's ideal is widespread among physicists. Yet, it is not always clear which kind of realism is entertained by authors adopting a realist interpretation of RT. Maybe in our post-modern times the idea that every observer may have his own reality is less objectionable, thus making Whiteheadean realism a possible alternative. Maybe it also is not always realized that a realist interpretation of RT must imply a fundamental relativity of this reality. In any case it seems to me that nowadays a realist interpretation is the more common one, both in textbooks on RT and in more foundational work (e.g., Friedman [10]).

For practical purposes this hardly poses any problem. Strengthening the interpretation of a measurement result from 'something I see' to 'something really existing' is innocent as long as we do not have independent means to probe the difference. John Bell's moving object *can* be considered as 'really' Lorentz contracted as long as no observation is at variance with such a supposition. Nevertheless I will discuss in section 3 a number of examples exhibiting some odd consequences of such a realist interpretation, indicating that maybe the abovementioned strengthening of the interpretation is more problematic than is often supposed.

The possibility of equating 'what is seen' to 'what is' hinges on a notion of ideality of the measurement procedure that may not be satisfied in actual practice. In an empiricist interpretation it is not possible to investigate this question directly because 'reality' is not thought to be described by the theory. It is possible, however, to compare different measurement procedures for equally prepared objects, and see whether they yield the same measurement results. In case results come out differently it is clear that some of the measurement procedures must be disturbing. A concrete example will be studied in section 4.

3. EXAMPLES

Twin paradox
The twin paradox is about two twin brothers, R and T, the former being permanently at rest in an inertial Lorentz frame of reference Σ (coordinates (t,x)), while the latter is travelling. After returning home T turns out to be younger than R. The problem has lost its paradoxical character since it is realized that T cannot be inertial throughout his journey, and, hence, is not equivalent with R. Yet, the problem remains instructive if not only the final result is considered but also the way it has come about.

For simplicity assuming that T's start and finish do not contribute to the difference, the inequivalence of R and T is restricted to a short interval during which T is reversing his direction of motion. During the larger part of the journey the twins are equivalent, however. This implies reciprocity, to the effect that there is a *mutual* clock retardation. Consequently, the problem cannot simply be solved by reference to 'moving clocks going slow'. The final result can only be obtained because there is no reciprocity during part of the journey. Only the reversal phase in which T, contrary to R, is non-inertial can contribute to this. In order to have agreement between the final readings it is necessary that, as seen by T, during his acceleration phase R's clock makes up for its lagging behind, and even takes a lead large enough to compensate for its being slow again during the journey back home. An impression of the different clock rates during different phases is given in Fig. 1.

In Fig. 1a) t^T and $\bar{t}^T$ are readings of T's clock as read off by R and T, respectively. If it is assumed that the rate of the moving clock is the one of the momentarily co-moving Lorentz frame (MCRF), we have

$$\bar{t}^T = \gamma^{-1} t^T, \ \gamma = (1 - v^2/c^2)^{-1/2},$$

v being the instantaneous velocity of T. A comparison of readings of R's clock by R and T (Fig. 1b) can be obtained, for instance, using a co-moving rigid reference frame $\bar{\Sigma}$ (coordinates $(\bar{t},\bar{x})$) constructed by pasting together MCRFs (cf. Møller [19], sect. 8.16):

$$\left.\begin{array}{l} x = c\int_0^{\bar{t}} dt \ \sinh\theta(t) + \bar{x}\cosh\theta(\bar{t}) \\ t = \int_0^{\bar{t}} dt \ \cosh\theta(t) + \frac{\bar{x}}{c}\sinh\theta(\bar{t}) \end{array}\right\}, \ \theta = \mathrm{arccosh}\gamma. \tag{1}$$

From this coordinate transformation the rates of R's clock can be compared by putting $x = 0$, thus yielding

$$t^R = \int_0^{\bar{t}^R} dt \ \cosh\theta(t) - \tanh\theta(\bar{t}^R)\int_0^{\bar{t}^R} dt \ \sinh\theta(t).$$

A consistent history for T can be obtained by taking

$$\theta(t) = \left\{\begin{array}{ll} gt/c, & 0 < t < t_1 \\ \theta = gt_1/c, & t_1 < t < t_2 \\ g(t_1+t_2-t)/c, & t_2 < t < t_2+2t_1 \\ -\theta, & t_2+2t_1 < t < t_1+2t_2 \\ g(t-2(t_1+t_2))/c, & t_1+2t_2 < t < 2(t_1+t_2) \end{array}\right. ,$$

yielding for the arrival time $\bar{t}^R = 2(t_1 + t_2)$ the relation

$$t^R = 2(t_2 - t_1)\cosh\theta + 4c/g\sinh\theta, \tag{2}$$

the last term in this expression being a correction due to the accelerations. By taking $\bar{x} = 0$ in eq. (1) an analogous reasoning can be applied to the moving clock, yielding

$$t^T = \int_0^{\bar{t}^T} dt\ \cosh\theta(t)$$

for the relation between R and T's observations. Although for this expression the final readings are consistent with the result (2), the time dependence of this relation is qualitatively similar to Fig. 1a).

In a realist interpretation of RT Fig. 1 could in principle be taken in the sense of Whitehead, each reference frame providing its own relative reality. Yet, from a physicists point of view this is not very attractive. The reason for this is the acausal behaviour of R's clock in the description of Fig. 1b). Evidently, if $\bar{t}^R$ would be the clock rate as it 'really' is relative to the reference frame $\bar{\Sigma}$ co-moving rigidly with T, then T's acceleration would have a strange non-local rate enhancing effect on R's clock. Such an effect would be contrary to the very spirit of RT, no local cause being available for an explanation of this effect. For this reason it would seem appropriate to try to find an alternative solution. Indeed, an empiricist interpretation offers better perspectives in this respect, because in this interpretation it is possible to accommodate the causal aspect of the correlation between T's acceleration and the enhancement of R's clock rate by reference to the measurement process. Maybe we should take seriously the often-used phraseology 'T is *seeing* a certain rate of R's clock', in the sense that the process of 'seeing' is a physical process of measurement liable to distort the input signal. The formalism of RT might describe just the output of the measurement process, i.e., the readings of the measuring apparata. Reciprocity can then be understood on the basis that, if both R and T are inertial, then their measurement procedures are equivalent, and data will be distorted in the same way. However, acceleration of T could influence his measurement procedure, thus causing him to 'see' an enhanced clock rate.

Radial free fall into a black hole

As a second example we consider a particle falling radially into a black hole. The solution of the geodesic equations is well-known ([20], p. 222). The important point is the qualitative difference, depicted in Fig. 2, of radial behaviour as described in terms of coordinate time ($r(t)$) and proper time ($r(\tau)$). In terms of t the object never crosses the Schwarzschild radius $r = 2m$, but it approaches this value of r asymptotically as t gets to infinity. In terms of proper time τ, on the contrary, the Schwarzschild radius is crossed in a finite time. This induces the question: 'What happens 'in reality'?' Does the object cross the Schwarzschild radius or does it not. This question could even have legal relevance if the object would be a space ship, and it should be decided whether life insurances should be paid to the astronauts' spouses.

Once again, in a realist interpretation of RT every reference frame could describe its own different reality. This solution might be favored by the Life Insurance Companies that, on the basis of this interpretation, could insist on the astronauts'

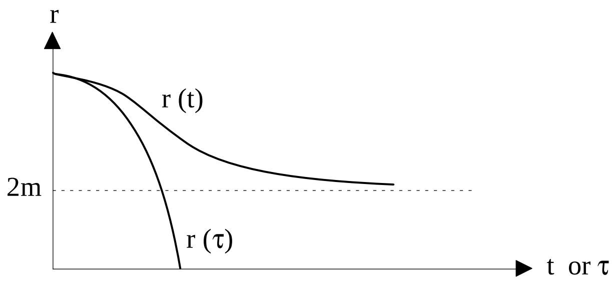

Figure 2: Radial coordinate as measured by a static and a co-moving observer.

happily living for ever, even though they only have a finite lifetime in their own reference frame. Once again, however, the empiricist interpretation offers a different perspective because of the possibility to insert a physical process of information transfer between object and observer. If the description by a static observer in terms of the Schwarzschild coordinates would just refer to information obtained by means of a causal signal transmitted from object to observer rather than as describing a factual state of affairs of the object itself, then it is possible to take into account retardation effects. In an empiricist interpretation of RT an event $(t, \mathbf{x})$ does not refer to the object being present at time t at space point $\mathbf{x}$, but to the readings of clocks and measuring rods induced by the causal signal. Near the Schwarzschild radius retardation effects may become very large because radial light velocity is vanishing there. Hence, the behaviour of $r(t)$ in Fig. 2 could be understood by taking into account the unbounded increase of signal transmission time as the object is approaching the Schwarzschild radius. The idea that an object never crosses the Schwarzschild radius is a consequence of a realist interpretation of the theory, neglecting the difference between 'what happens' and 'what is seen'.

Length measurement in special relativity
Let the inertial observer T (Lorentz frame $\bar{\Sigma}$, coordinates $(\bar{t}, \bar{x})$) measure the length of some object, at rest in R's inertial frame Σ (coordinates (t, x)), by simultaneously marking off its endpoints on his own measuring rod.

It is interesting to describe this marking process in the object's rest frame. This can simply be done using the Lorentz transformation

$$x = \gamma(\bar{x} - v\bar{t}),\ t = \gamma(\bar{t} - v\bar{x}/c^2). \tag{3}$$

The marking events P_A and P_B of the endpoints are given by $(\bar{t}_A, \bar{x}_A)$ and $(\bar{t}_B, \bar{x}_B)$, respectively, with $\bar{t}_A = \bar{t}_B$. Now (3) holds for both the A and B events. From this the Lorentz contraction follows directly as $\bar{x}_A - \bar{x}_B = \gamma^{-1}(x_A - x_B)$, demonstrating the appropriateness of this measurement procedure within the domain of application of RT. Yet, it is somewhat surprising to see how this procedure looks as seen from Σ.

The surprise is *not* that $t_A \neq t_B$ (relativity of simultaneity). For the procedure of Fig. 3 it can easily be verified that $t_B > t_A$, meaning that in R's description point A is marked off first on T's measuring rod. What is important is that during the time interval (t_A, t_B) T's measuring rod is moving with velocity $-v$ with respect to

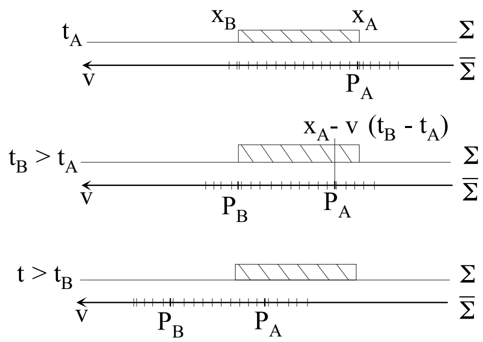

Figure 3: Measurement procedure for the length of a moving object, performed in $\bar{\Sigma}$, as seen from Σ by observers co-moving with the object.

R and his object. This implies that the point P_A of T's measuring rod at time t_B is at position $x_A - v(t_B - t_A)$.

If R would entertain a realist interpretation of RT, this would be a rather alarming observation, since the interval (P_A, P_B) finally marked off by T did not have a complete overlap with R's object at any time of R's description. R's description of T's measurement procedure would not be distinguishable from the description of the acts of a fraudulent surveyor who, on measuring a piece of land, allows his measuring rod to slip while walking from A to B. On the basis of a realist interpretation of his description R would have to distrust T's measurement result. Also here an empiricist interpretation could do justice to T's fair intentions, however: what R sees need not be the same as what is really happening.

4. IDEAL AND NON-IDEAL MEASUREMENTS IN RT

In a realist interpretation of a physical theory a measurement is thought to determine the value of some property of the object. For instance, the outcome of a length measurement is considered as a measure of the spatial extension of the object. The measurement may be non-ideal, yielding a value to some extent differing from the 'real' value. In an empiricist interpretation the theory is not supposed to refer to any 'real' value. Here only measurement procedures are thought to be compared by the theory. Thus, it is possible to interpret some measurement procedure as a non-ideal version of another procedure. For instance, a length measurement by a moving observer might be interpreted as a non-ideal version of a length measurement by a co-moving observer (and vice versa). Whereas in QM it has been possible to develop criteria allowing a partial ordering among its observables (i.e., measurement procedures) [1], in RT as yet we do not have means to tell which measurement procedure is the better one. Hence, strictly speaking we are not entitled to believe that the length measurement by a co-moving observer is more ideal than the one performed by a moving observer.

Measurement in RT is governed by the metric tensor $g_{\alpha\beta}$. In particular by the requirement that its covariant derivative vanish,

$$\nabla g = 0, \tag{4}$$

particular relations are induced between length and time measurements in different points of space-time, or between descriptions of the same measurement in different

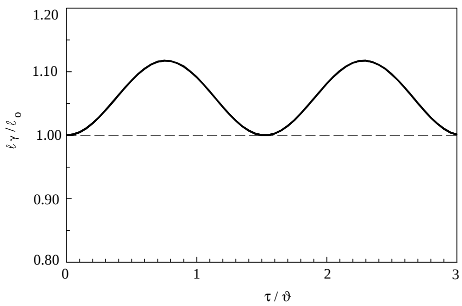

Figure 4: Behaviour of the measuring rod as seen from the MCRF: $\ell\gamma/\ell_0$ vs. τ/θ (with $k\theta/m^2 = 9$). The broken curve shows the behaviour of a Lorentz contracted rod, $\ell/\ell_0 = 1/\gamma$.

coordinate frames (such as the relations of Lorentz contraction and time dilation between length and time measurements in different Lorentz frames in special RT). It is the axiomatically imposed condition (4) that is governing the relation between the structure of space-time (the affine connections) and the metric tensor.

We can now ask the same question as we did in QM, viz., do realistic measuring instruments really satisfy the kinematic prescription (4), or is it just a condition singling out a specific kind of measurements that are acceptable within the domain of RT? It is well-known [21] that the condition (4) specifies a length measurement as a procedure using a (Born) rigid measuring rod each element of which experiencing a Lorentz contraction as its velocity is changed. One possibility for an alternative length measurement procedure might be given by a system of two non-interacting particles having identical acceleration programmes in some Lorentz frame Σ ([17], chapt. 9). Since the world lines of these particles can be transformed into each other by a simple space translation, their distance as seen from Σ remains constant, independently of their velocity with respect to Σ. Hence, there is no Lorentz contraction. Indeed, in an MCRF the distance between the particles is considerably increased as compared to the distance seen in the fixed frame. It is possible, at least in principle, to choose such a system as a measuring rod, and perform length measurements with it on moving objects. However, the measurement outcomes obtained with this instrument would show large deviations from the predictions of RT: in the fixed Lorentz frame Σ the instrument is behaving like a classical measuring rod even though the two particles satisfy relativistic equations of motion.

A better approximation of a relativistic measuring rod is obtained if the two particles are connected by an ideal massless spring [22] with spring constant k. Under the condition of small relative velocity the relativistic equations of motion can then be written as

$$\begin{aligned} m\tfrac{dv_1}{dt} &= (1 - {v_1}^2/c^2)^{3/2}\,[F - k(\ell_0 - \ell\gamma)], \\ m\tfrac{dv_2}{dt} &= (1 - {v_2}^2/c^2)^{3/2}\,[F + k(\ell_0 - \ell\gamma)], \end{aligned}$$

in which $\ell = x_2 - x_1$ and ℓ_0 is the string's proper length. F is a constant force exerted on each of the particles. Under the assumption $v_1 \approx v_2$ an MCRF can be defined for the two-particle system, and, hence, a proper time τ. We then find as an approximate solution of the equations of motion

$$\ell\gamma = \ell_0[1 + \frac{(1 - \cos\omega\tau)}{\omega^2\theta^2}],\ \omega = (2k/m - \theta^{-2})^{1/2},\ \theta = \frac{mc}{F} \qquad (5)$$

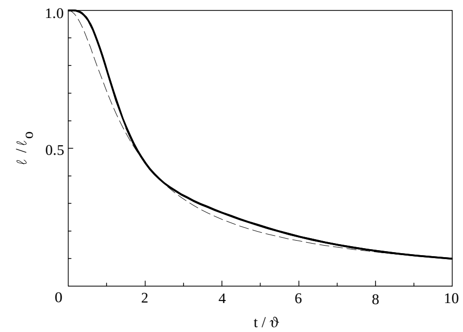

Figure 5: Behaviour of the measuring rod as seen by an observer in Σ: ℓ/ℓ_0 vs. t/θ.

(cf. Fig. 4), the corresponding behaviour as a function of t being given in Fig. 5.

From (5) and Figs 4 and 5 it is evident that a length measurement performed with this object in general will not yield the result prescribed by the Lorentz contraction (represented by the broken lines). Only in the limit $k \to \infty$ the relativistic measurement prescription (4) is satisfied. It, of course, is very satisfactory that this limit coincides with Born's requirement that a relativistic measuring rod be a rigid body. Evidently, the kinematic prescription (4) of relativistic measurement is not at variance with a relativistic dynamic description of the measurement process.

5. CONCLUSIONS

Although Lorentz contraction (and time dilation [22]) are consistent with a dynamic description of the behaviour of length and time measuring instruments under slow transport, it is also clear that this is only true in an unphysical 'ideal' limit. Real measuring rods at best satisfy the requirements (4) to a certain approximation. It even is possible to choose measurement procedures yielding vastly different results. This induces the question why it should be necessary to restrict to measurement procedures satisfying (4). The tentative answer that such measurements would yield the more faithful information about reality could hardly be given in an empiricist interpretation. Lorentz contraction and time dilation might themselves be consequences of a certain way of observation, retardation effects being taken into account in the relativistic formalism.

Retardation effects have been discussed in the past after a demonstration by Penrose [23] that the circumference of a moving sphere would not show up a Lorentz contraction in the direction of motion if the time differences of optical signals from different points of the sphere are duly taken into account. In these discussions (Terrell [24], Weinstein [25], Scott and Viner [26], Mathews and Lakshmanan [27]) it is generally taken for granted that relativistic space-time coordinates describe the 'real' shape of the object whereas taking into account retardation effects yields only an 'apparent' shape, the latter being endorsed by the fact that different photographic techniques may yield different 'apparent' shapes. In this respect by McGill [28] the notion of a *non-ideal* measurement is used to indicate a measurement not unambiguously determining the apparent shape.

In an empiricist interpretation of RT also the shape described by the relativistic space-time coordinates must be interpreted as an *apparent* shape. These data must also be interpreted as results of certain measurements, be it that no general operationalization of these measurements has been given up till now. It is clear both from the calculations in sect. 4 and from the abovementioned discussion that there exist many observation procedures not yielding outcomes that correspond with an objective description of the object. Like in QM this provokes the question why the 'canonical' relativistic measurement procedures would yield a better description of reality. Indeed, although we might have some reason to suppose that a measurement performed in a co-moving frame yields a more faithful picture of its immediate neighbourhood, measurements performed on moving objects seem to be liable to non-ideality also if they exactly reproduce the predictions of RT.

I can agree with Bub [29] that our most fundamental physical theories should be not simply about measurements, but about the behaviour of physical systems. However, we should realize that we get our information about this behaviour solely by means of our observations, i.e., through the mirror of our measuring devices. If this mirror would happen to be not a flat mirror, the picture of reality we would obtain would be a distorted picture. Empirical adequacy of our theories would imply that these theories yield a representation of this distorted picture in the first place. Only if the complete set of our measurement procedures would be comparable to a flat mirror, Bub would be right in supposing that our theories give us direct information about the physical systems themselves. It is, however, not very likely that this is the situation we are in. It seems that both in QM and in RT the situation is, indeed, comparable to a non-flat mirror. As far as the theories remain empirically adequate also in this situation we will have to allow for a correction of this distortion in order to obtain a true picture of reality.

In order that a unification of QM and RT be possible, it seems essential that both theories are interpreted in a similar way. According to Ehlers and Schäfer [30] the problems met in the attempt at unification are caused by the double role of space-time metric which, on the one hand, is interpreted as a (tensorial) gravitational potential, and, on the other hand, enters in the kinematical prescriptions governing measurements of spatial distance and time. An analogous double role is played by the quantum mechanical observable in many interpretations of QM. From a methodological point of view such a double role is undesirable. It seems to me that an empiricist interpretation of both theories can provide a consistent picture in which both the quantum mechanical observable and space-time metric are just kinematical prescriptions defining measurement procedures valid within the domains of application of quantum mechanics and RT, respectively. To what extent observable and metric can be considered as properties of an underlying reality does not seem to be a question that can be answered on the level of either QM or RT. However, as we have seen above, paradoxes may arise if such a realist interpretation is applied too easily.

Recently by Debs and Redhead [31] a new approach of the twin paradox was presented, based on the conventionality of simultaneity. It seems to me that, indeed, conventionality may play an important role in understanding RT. Which measurement procedures an observer is willing to rely on evidently is a matter of convention. A choice for measurement procedures satisfying the kinematic prescription (4) corresponds to a very specific convention of comparing measurement procedures for length and time. Other choices are possible, however. Contrary to Debs and Redhead, I do not think, however, that reference to conventionality would be a final solution, and would put an end to questions like the twin paradox. Conventions should be based on objective facts, i.e., on the choice of well-defined measurement procedures. Thus, the difference between Greenwich time and Middle European time is a matter of convention. Yet, it is of extreme importance to know how to handle my watch when arriving in London after having left Eindhoven. An operational approach of relativity theory, encompassing the physical operations of length and time measurement the theory is based on, seems to be equally desirable.

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