OPTICAL EXPERIMENTS ILLUSTRATING THE SIGNIFICANCE OF THE BELL INEQUALITIES

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Recently the possibility of generating squeezed states of light has provided for an important new tool in studying questions that are of crucial importance for the foundations of quantum mechanics. Squeezed states, being states of the electromagnetic field without a classical analogue, seem particularly suited for studying effects that are typically quantum mechanical. For this reason it is not surprising that they have been applied in tests of the Heisenberg uncertainty relation[1] and of the Bell inequalities (BI)[2]. In the latter experiment, which was performed by Ou and Mandel[2], the two–photon character of squeezed states generated in parametric downconversion is used in order to obtain a correlated state of the electromagnetic field violating the BI. This was verified in Einstein–Podolsky–Rosen (EPR)–like experiments in which the joint probability $P(\theta_1,\theta_2)$ was measured of one photon being polarized in the direction θ_1 and the other one in the direction θ_2. In order to test the BI the experiment was repeated for different values of θ_1 and θ_2, yielding $-1 \leq P(\theta_1,\theta_2) - P(\theta_1,\theta'_2) + P(\theta'_1,\theta_2) + P(\theta'_1,\theta'_2) - P(\theta'_1) - P(\theta_2) \leq 0$ if the BI are satisfied.

Violation of the BI is often interpreted as violation of Einstein locality, to the effect that in the EPR–like experiments the two (polarization) measurements would be able to influence each other, even if they are performed in causally disjoint regions of spacetime, and, consequently, correspond with commuting observables. By de Muynck and Abu–Zeid[3] an alternative interpretation of the BI was discussed, connecting the violation of the BI with the *in*compatibility of the observables that are involved, rather than with the locality issue. If it would be possible to measure simultaneously the four (noncommuting) observables that are involved, then the existence of a quadrivariate joint probability distribution (JPD) would guarantee the BI to be satisfied[4], even if they are violated when the measurements are done in the EPR–like way.

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The theory of the simultaneous or joint measurement of noncommuting observables has been developed on the basis of the concept of observables as positive operatorvalued measures (POVM)[5,6]. The POVM $\{\mathbf{M}_{ij}\}$ represents a joint measurement of the POVMs $\{\sum_i \mathbf{M}_{ij}\}$ and $\{\sum_j \mathbf{M}_{ij}\}$, the latter POVMs reproducing the marginal probabilities of the JPD $\mathrm{Tr}\boldsymbol{\rho}\, \mathbf{M}_{ij}$.

Busch[7] considers the joint measurement of two polarization observables in (incompatible) directions θ_1 and θ'_1. By a beam splitter (transparancy γ) the incoming beam is split, and polarizations in directions θ_1 and θ'_1 are measured in the transmitted and the reflected beam, respectively. According to Busch the corresponding detection probabilities in the joint measurement are represented by the positive operators $\gamma\mathbf{E}(\theta_1 = +)$ and $(1-\gamma)\mathbf{E}(\theta'_1 = +)$, respectively, $\mathbf{E}(\theta_1 = +)$ and $\mathbf{E}(\theta'_1 = +)$ being the usual projection operators. Defining the POVM $\{\mathbf{M}_{11} = 0,\ \mathbf{M}_{12} = \gamma\mathbf{E}(\theta_1 = +),\ \mathbf{M}_{21} = (1-\gamma)\ \mathbf{E}(\theta'_1 = +), \mathbf{M}_{22} = \mathbf{I} - \mathbf{M}_{11} - \mathbf{M}_{12} - \mathbf{M}_{21}\}$, we obtain as its marginals the POVMs $\{\sum_j \mathbf{M}_{ij}\} = \{\ \sum_{k=\pm} \lambda_{ik}\mathbf{E}(\theta_1 = k)\}$ and $\{\sum_i \mathbf{M}_{ij}\} = \{\ \sum_{k=\pm} \mu_{jk}\mathbf{E}(\theta'_1 = k)\}$, the (stochastic) matrices (λ_{ik}) and (μ_{ik}) being defined according to

$$(\lambda_{ik}) = \begin{bmatrix}\gamma & 0\\ 1-\gamma & 1\end{bmatrix}, \qquad (\mu_{jk}) = \begin{bmatrix}1-\gamma & 0\\ \gamma & 1\end{bmatrix}.$$

These POVMs can be interpreted as describing the results of nonideal measurements of the polarization observables in the directions θ_1 and θ'_1, respectively[8]. Note, however, that the ideal information can be obtained from the nonideal one by inverting the two matrices. As a matter of fact, it can easily be verified that the relation $\mathbf{W}_{kl} = \sum_{ij}(\lambda^{-1})_{ki}(\mu^{-1})_{lj}\mathbf{M}_{ij}$ yields the operator valued (Wigner) measure $\{\mathbf{W}_{kl}\} = \{0, \mathbf{E}(\theta_1{=}+), \mathbf{E}(\theta'_1 = +), \mathbf{I} - \mathbf{E}(\theta_1{=}+) - \mathbf{E}(\theta'_2{=}+)\}$, which has the spectral measures $\{\mathbf{E}(\theta_1{=}\pm)\}$ and $\{\mathbf{E}(\theta'_1{=}\pm)\}$ as its marginals.

Quantum Measurements Satisfying the Bell Inequalities

In this contribution we want to discuss optical experiments in which the measurement results can be changed from violating the BI to satisfying these, by changing the measurement arrangement from EPR–like to a joint measurement of the four observables, without changing the state function. One possibility is to modify the experiment by Ou and Mandel[2] in the way indicated in fig. 1, by replacing in each arm of the interferometer the single–observable measurement arrangement by Busch's setup for the joint measurement of two polarization observables. In this experiment coincidences are measured between the four detectors D_1, D'_1, D_2 and D'_2, taking into account only those events in which one detector on each side (1 or 2) counts a photon. This experiment is described by a POVM that is a direct product $\{\mathbf{M}^{(1)}_{ij}\ \mathbf{M}^{(2)}_{kl}\}$ of two

POVMs $\{M_{ij}^{(m)}\}$, m = 1,2, as defined above. Hence, there exists a quadrivariate JPD $P(\theta_1,\theta'_1,\theta_2,\theta'_2)$ having as marginals the bivariate JPDs $P'(\theta_1,\theta_2)$ etc., differing from the EPR probabilities $P(\theta_1,\theta_2)$ because θ_i is measured jointly with the noncommuting polarization observable θ'_i. It is easily verified that for transparancies γ_1 and γ_2 of the two beam splitters BS_1 and BS_2 the bivariate JPDs are related according to $P'(\theta_1,\theta_2) = \gamma_1\gamma_2\, P(\theta_1,\theta_2)$, $P'(\theta_1,\theta'_2) = \gamma_1(1-\gamma_2)P(\theta_1,\theta'_2)$, etc., enabling a direct proof of the validity of the BI for the JPD's $P'(\theta_1,\theta_2)$. It must be stressed that the transition from data $P'(\theta_1,\theta_2)$ satisfying the BI to data $P(\theta_1,\theta_2)$ (possibly) violating the BI is brought about by replacing the POVM $\{M_{ij}^{(m)}\}$ with the Wigner measure $\{W_{ij}^{(m)}\}$ in each arm of the interferometer (m = 1,2) separately. This suggests that the importance of the BI is connected with the correlations between the two incompatible observables in each arm separately, rather than with distant correlations between compatible observables.

Optical experiments can also be used to explore the implications for the BI of the canonical commution relations between position and momentum. By Yuen[9] it was realized that optical measurements like optical homodyning and heterodyning can be interpreted as simultaneous (nonideal) joint measurements of these (noncommuting) observables. One example is eight–port optical homodyning[10,11], to be interpreted as a simultaneous phase and amplitude measurement. Consider the eight–port homodyne detector of fig.2, in which L is a monochromatic local oscillator in a coherent state, and the transparancies of the mirrors are as indicated. There is an extra $\pi/2$ phase shift between the input signal S and detector D_1. Then, for monochromatic input signal, having the same frequency as L, we find the following POVM for the probabilities of the output signals F_1 and F_2:

$$M(dF_1,dF_2) = \frac{dF_1dF_2}{\pi\sigma_1\sigma_2} \int \frac{d^2\beta}{\pi}\, |\beta;\lambda\rangle\langle\beta;\lambda|\, e^{-\frac{(F_1-\beta_1\sqrt{2\lambda})^2}{\sigma_1^2}-\frac{(F_2-\beta_2\sqrt{2/\lambda})^2}{\sigma_2^2}},$$

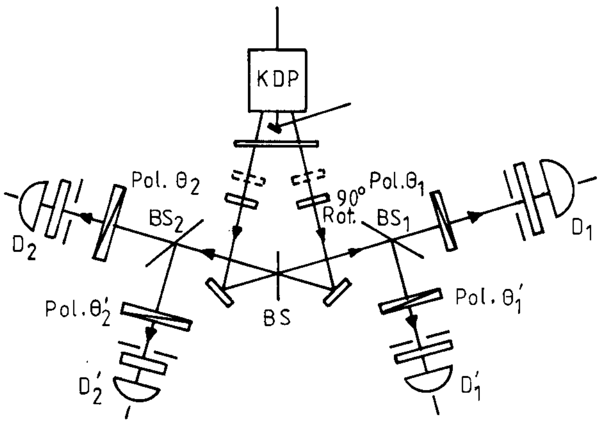

Fig.1. Joint measurement of four polarization observables.

$$\sigma_1^2 = \frac{1}{(1-\kappa)}\left(\frac{1}{4\eta\epsilon_1(1-\epsilon_1)} - 1\right),\quad \sigma_2^2 = \frac{1}{\kappa}\left(\frac{1}{4\eta\epsilon_2(1-\epsilon_2)} - 1\right),\quad \lambda = \frac{\kappa}{1-\kappa},\quad \beta = \beta_1 + i\beta_2,$$

in which η is the quantum efficiency of the detectors, and $|\beta;\lambda>$ is a squeezed state[12] $|\mu\nu\beta>$ with $\mu = (1+\lambda)/2\surd\lambda$, $\nu = (1-\lambda)/2\surd\lambda$ (this POVM was essentially already derived in ref.13). The marginals of this POVM are

$$\mathbf{M}_1(dF_1) = \frac{dF_1}{\delta_1\sqrt{\pi}} \int_{-\infty}^{+\infty} dq\; e^{-\frac{(F_1-q)^2}{\delta_1^2}} |q><q|,$$

$$\mathbf{M}_2(dF_2) = \frac{dF_2}{\delta_2\sqrt{\pi}} \int_{-\infty}^{+\infty} dp\; e^{-\frac{(F_2-p)^2}{\delta_2^2}} |p><p|,$$

$$\delta_1^2 = \frac{1}{4(1-\kappa)\eta\epsilon_1(1-\epsilon_1)} - 1,\quad \delta_2^2 = \frac{1}{4\kappa\eta\epsilon_2(1-\epsilon_2)} - 1,$$

thus demonstrating that eight–port homodyne detection can be interpreted as a joint nonideal measurement of the incompatible observables $q = (a+a^\dagger)/\surd 2$, $p = (a-a^\dagger)/i\surd 2$.

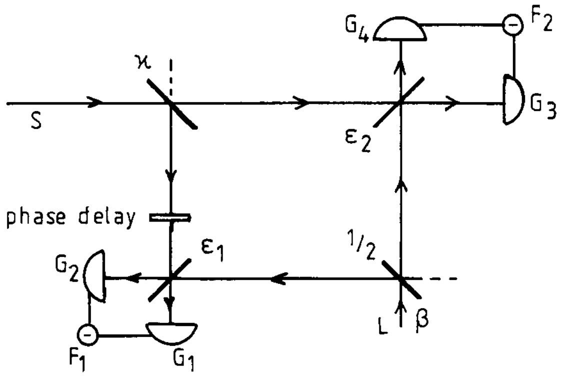

Fig.2. Eight-port optical homodyning as a joint measurement of q and p.

As in the case of the polarization measurements it is possible here to invert the relation between the POVM $\{\mathbf{M}_i(dF_i)\}$ and the spectral measure, and define a Wigner measure analogous to $\{\mathbf{W}_{kl}\}$. This Wigner measure can be represented, using coherent states $|\beta>$, by

$$W(q,p) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} dk \int_{-\infty}^{+\infty} dk' \int \frac{d^2\beta}{\pi} |\beta\rangle\langle\beta| e^{\frac{1}{4}(k^2+k'^2)+ik(q-\beta_1\sqrt{2})+ik'(p-\beta_2\sqrt{2})} .$$

It can easily be shown that Trρ**W**(q,p) coincides with the usual Wigner distribution[14], thus exhibiting that also here the usual quantum mechanical data can be obtained from the joint measurement.

If in the measurement arrangement of fig.1 in each arm of the interferometer the whole setup of beam splitter, polarizers and detectors is replaced by an eight–port homodyne detector (taking care, as is done in ref. 1, to extract the local oscillator beams from the same laser that excites the optical parametric oscillator), the experiment can be interpreted as a joint nonideal measurement for the observables q_1,p_1,q_2 and p_2. Once again the result of the simultaneous measurement has to satisfy the BI, even if the state is nonclassical, and is it possible to calculate the exact data (in the non–classical case violating the BI) from the Wigner measure.

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