

**OBJECTIVITY, NONLOCALITY, AND THE BELL INEQUALITIES**

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Received 12 October 1982

Revised manuscript received 21 December 1982

It is argued that the Bell inequalities are not a specific feature of *local* hidden variables theories, but obtain both for local and nonlocal theories that are objectivistic. A nonobjectivistic local theory is constructed reproducing the quantum mechanical correlations obtained in realisations of the Einstein-Podolsky-Rosen-Bohm experiment.

In this note the notion of objectivity signifies the possibility of a description of a physical object as an *isolated* system. As is well known, Bohr most forcefully denied the possibility of an objectivistic interpretation of the predictions of the quantum mechanical formalism. In his opinion, by the macroscopic measurement arrangement conditions are fixed which "constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached ..." [1]. Bohr's answer to the challenge of Einstein, Podolsky and Rosen [2] essentially has a nonobjectivistic character. According to Bohr the expression "without in any way disturbing a system" [2] is ambiguous, because it does not take into account "the influence (of the measuring procedure) on the very conditions which define the possible types of predictions regarding the future behavior of the system" [1].

It is remarkable that in present-day formulations and interpretations of quantum mechanics so little has survived of Bohr's nonobjectivism. Apart from a rather modest number of papers devoted to measurement theory, measuring instrument and measurement interaction are left out of consideration. An analogous statement can be made with respect to the attempts to devise realistic theories underlying quantum mechanics: so-called contextualistic theories play a very minor part here. Most hidden variables theories are, either explicitly or implicitly, of the objectivistic type. This is implemented by taking the probability distribution

of the hidden variables independent of the measurement context. Contextualistic hidden variables theories would seem, however, to correspond better to Bohr's understanding of microphysical reality [3]. That this line has not been pursued more intensively may be due to the fact that an explanation of the EPR-paradox in this way is often interpreted as demanding a *nonlocal* influence of the measurement arrangement on the state of a distant object system that is *not* in "physical" interaction with the instrument ("no question of a mechanical disturbance" [1]).

The idea of nonlocality of the microphysical world has pervaded the discussion ever since the derivation of the Bell inequalities [4]. It should be stressed that, since the Bell inequalities are derived from an *objectivistic* hidden variables theory, this kind of nonlocality is different from the nonlocality attributed to Bohr's solution of the EPR-paradox. Whereas in the latter the object and measuring instrument are thought to form an inseparable whole (nonobjectivity), in the former inseparability [5] is attributed to the (parts of the) object system and is thought of as an objective property of the object.

It is the purpose of this note to draw attention to the facts that

- (i) the Bell inequalities do not imply nonlocality of the microphysical world;
- (ii) the quantum mechanical correlation data involved in, but not satisfying, the Bell inequalities, can be reproduced by a nonobjectivistic (contextualistic) *local* hidden variables theory.

The first assertion is based mainly on the recent work of Fine [6,7], in which the Bell inequalities are derived from the existence of well-defined, compatible joint distributions for all pairs and triples of commuting and noncommuting observables. This derivation is quite general. More specifically, it is independent of the question whether the hidden variables are local or nonlocal. It merely hinges on the assumed existence of joint distributions, which does not presuppose any form of locality. As a matter of fact, also in other derivations of the Bell inequalities there is no explicit reference to the distance between the regions of the measurements.

Fine's derivation shows that the Bell inequalities obtain for *any* hv theory having the abovementioned joint distributions. It was shown in ref. [8] that, if such joint distributions are interpreted in the objectivistic realistic sense which characterises the so-called ignorance interpretation of quantum mechanics (joint probability distributions of the first kind [8]), then the theory should deviate from quantum mechanics. It appears that the Bell inequalities are just another evidence of the incompatibility of quantum mechanics with objectivistic hidden variables theories of this kind, be they local or nonlocal. Since in Fine's hv theory *single* probabilities are required to have the corresponding quantum mechanical values, his theory is in accordance with the ignorance interpretation of quantum mechanics. Hence, it is this kind of hv theory that is withdrawn by the Bell inequalities from the set of those hv theories which are consistent with quantum mechanics. Consequently, since in Fine's theory no reference needs to be made to locality or nonlocality of the hidden variables, the Bell inequalities do not rule out the possibility of an objectivistic, *local* hidden variables theory which does *not* correspond to the ignorance interpretation (e.g. by dropping the requirement that the subdivision of an ensemble should be governed by the projection postulate).

In order to demonstrate the second assertion I consider a model in which the hidden variables are represented by a stochastic field  $\lambda(\mathbf{r})$ , and the expectation values of the magnitudes by functional integrals of the form [9]

$$\langle A \rangle = \int_{\Lambda} A(\lambda) \rho(\lambda) \mathcal{D}(\lambda), \quad (1)$$

in which  $\rho(\lambda)$  is a density functional. In a contextualistic theory the density functional is dependent on the measurement arrangement. Hence, if  $\rho(\lambda)$  is the density functional if no measurement is made, and  $T_A$  is an operator representing the influence of the  $A$ -meter, we have, instead of (1),

$$\langle A \rangle = \int_{\Lambda} A(\lambda) T_A \rho(\lambda) \mathcal{D}(\lambda), \quad (2)$$

$$\int_{\Lambda} T_A \rho(\lambda) \mathcal{D}(\lambda) = \int_{\Lambda} \rho(\lambda) \mathcal{D}(\lambda) = 1. \quad (3)$$

If  $A$  corresponds to a quantum mechanical observable, having (possibly degenerate) eigenvalues  $a_i$  and eigenvectors  $|\alpha_{ij}\rangle$ , and  $A(\lambda) = a_i, \lambda \in \Lambda_i$ , the equivalence of (2) with the quantum mechanical outcomes requires that

$$\int_{\Lambda_i} T_A \rho(\lambda) \mathcal{D}(\lambda) = \sum_j \langle \alpha_{ij} | \rho | \alpha_{ij} \rangle, \quad (4)$$

at the right-hand side of which  $\rho$  is the density operator of the quantum mechanical state, which is represented by  $\rho(\lambda)$  in the hv theory. Since now  $T_A \rho(\lambda)$  may differ from  $T_{A'} \rho(\lambda)$  if  $[A, A'] \neq 0$ , it is possible to arrange the consistency of quantum mechanics with the contextualistic hv theory. If  $A$  and  $B$  are measured simultaneously ( $[A, B] = 0$ ), and  $|\alpha_{ij}\rangle$  are the simultaneous eigenvectors of  $A$  and  $B$ , (4) can be generalized according to

$$\int_{\Lambda_{ij}} T_{AB} \rho(\lambda) \mathcal{D}(\lambda) = \langle \alpha_{ij} | \rho | \alpha_{ij} \rangle, \quad (5)$$

$\Lambda_{ij}$  being that part of  $\Lambda$  in which  $A(\lambda) = a_i$  and  $B(\lambda) = b_j$ . Also,

$$\langle AB \rangle = \int A(\lambda) B(\lambda) T_{AB} \rho(\lambda) \mathcal{D}(\lambda). \quad (6)$$

We now specialize to *local* measurements  $A$  and  $B$ , performed in disjoint regions of space. It seems reasonable to assume the following to be true for the local measurements:

(i)  $A(\lambda)$  does not depend on the values which  $\lambda(\mathbf{r})$  has outside the region of the  $A$ -measurement, and analogously for  $B(\lambda)$ ;

(ii)  $T_{AB} = T_A T_B = T_B T_A$ , and  $T_A (T_B)$  does not

influence the probability density  $\rho(\lambda)$  outside the region of the  $A$ - ( $B$ -) measurement.

Defining

$$\lambda_A(\mathbf{r}) = \chi_A(\mathbf{r})\lambda(\mathbf{r}), \quad \lambda_B(\mathbf{r}) = \chi_B(\mathbf{r})\lambda(\mathbf{r}),$$

$$\lambda_{\overline{AB}}(\mathbf{r}) = \chi_{\overline{AB}}(\mathbf{r})\lambda(\mathbf{r}), \quad (7)$$

$\chi_A(\mathbf{r})$ ,  $\chi_B(\mathbf{r})$  and  $\chi_{\overline{AB}}(\mathbf{r})$  being the characteristic functions of the  $A$ - and  $B$ -regions and of the complement  $\overline{AB}$  of their join, respectively, we may carry out the functional integration according to  $\mathcal{D}(\lambda) = \mathcal{D}(\lambda_A) \times \mathcal{D}(\lambda_B) \mathcal{D}(\lambda_{\overline{AB}})$ . Then, with

$$\rho_{AB}(\lambda_A, \lambda_B) = \int \rho(\lambda) \mathcal{D}(\lambda_{\overline{AB}}), \quad (8)$$

we obtain

$$\langle AB \rangle = \int A(\lambda_A) B(\lambda_B) T_A T_B \rho_{AB}(\lambda_A, \lambda_B) \mathcal{D}(\lambda_A) \mathcal{D}(\lambda_B). \quad (9)$$

The problem of the Bell inequalities amounts to the question whether the correlations (9) are equal to the quantum mechanical correlations  $\text{Tr} \rho_{AB}$  for *all* local observables of the  $A$ - and  $B$ -regions. This is equivalent to the possibility of the equality [cf. (5)]

$$\int_{\Lambda_{ij}^{AB}} T_A T_B \rho_{AB}(\lambda_A, \lambda_B) \mathcal{D}(\lambda_A) \mathcal{D}(\lambda_B) = \langle \alpha_{ij} | \rho | \alpha_{ij} \rangle, \quad (10)$$

$\Lambda_{ij}^{AB} = \Lambda_i^A \times \Lambda_j^B$  being the restriction of  $\Lambda_{ij}$  to the  $AB$ -region. However, (10) can easily be fulfilled if (4) obtains for all local observables. In fact, for the local observable  $A$  we get for the left hand side of (4)

$$\int_{\Lambda_i} T_A \rho(\lambda) \mathcal{D}(\lambda) = \int_{\Lambda_i^A} T_A \rho_A(\lambda_A) \mathcal{D}(\lambda_A)$$

$$= \int_{\Lambda_i^A \times \Lambda^B} T_A \rho_{AB}(\lambda_A, \lambda_B) \mathcal{D}(\lambda_A) \mathcal{D}(\lambda_B), \quad (11)$$

with

$$\rho_A(\lambda_A) = \int \rho(\lambda) \mathcal{D}(\lambda_B) \mathcal{D}(\lambda_{\overline{AB}})$$

$$= \int \rho_{AB}(\lambda_A, \lambda_B) \mathcal{D}(\lambda_B). \quad (12)$$

Analogous expressions can be obtained for  $B$ . Hence the question boils down to the existence of a probability function  $\rho_{AB}(\lambda_A, \lambda_B)$  for which (10) is fulfilled if

$$\int_{\Lambda_i^A \times \Lambda^B} T_A \rho_{AB}(\lambda_A, \lambda_B) \mathcal{D}(\lambda_A) \mathcal{D}(\lambda_B) = \sum_j \langle \alpha_{ij} | \rho | \alpha_{ij} \rangle, \quad (13)$$

and

$$\int_{\Lambda^A \times \Lambda_j^B} T_B \rho_{AB}(\lambda_A, \lambda_B) \mathcal{D}(\lambda_A) \mathcal{D}(\lambda_B) = \sum_i \langle \alpha_{ij} | \rho | \alpha_{ij} \rangle. \quad (14)$$

From the assumption that the local  $A$ -measurement is not influenced by the presence of the  $B$ -meter (and vice versa), it follows that

$$\int T_A \rho_{AB}(\lambda_A, \lambda_B) \mathcal{D}(\lambda_B) = \int T_A T_B \rho_{AB}(\lambda_A, \lambda_B) \mathcal{D}(\lambda_B), \quad (15)$$

and equivalently for  $B$ . From (15) it is seen that (13) is just one of the two marginal relations of (10), and equivalently for (14). Then, the existence of a probability distribution  $T_A T_B \rho_{AB}(\lambda_A, \lambda_B)$  satisfying (10) if (13) and (14) are fulfilled, follows from classical probability theory, since for  $[A, B] = 0$  the  $\langle \alpha_{ij} | \rho | \alpha_{ij} \rangle$  are classical probabilities. It is clear that the Bell inequalities can be avoided since the probability distribution changes when a different measurement arrangement is chosen. This completes the demonstration of my second assertion.

It was concluded in ref. [10] that "only two loopholes remain open for advocates of realistic theories without action at a distance", viz. the low efficiencies of the detectors, and the static character of all previous experiments. In this note a third loophole is considered, and demonstrated to be a reasonable possibility to avoid too hasty conclusions with respect to nonlocality of the microphysical world, drawn from violations of the Bell inequalities in experimental realisations of the

Einstein–Podolsky–Rosen–Bohm experiment. Admittedly, the results obtained here do not exclude the possibility that, on a microscopic level, reality is nonlocal. It only demonstrates that such a nonlocality, if it exists, need not be revealed by experiments of the above-mentioned type. This conclusion is not obviated if, like in the Aspect experiment proposed in ref. [11], the measuring arrangement is switched. Also here, violation of the Bell inequalities does not necessarily imply nonlocality, because this violation can be attributed to the local influence of the measuring instruments.

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