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On the Significance of the Bell Inequalities for the Locality Problem in Different Realistic Interpretations of Quantum Mechanics

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Abstract. The relation between the Bell inequalities, locality and the existence of joint probability distributions is discussed in different realist interpretations of quantum mechanics. We distinguish four realist interpretations, viz., the objectivistic one, the contextualistic one, the strictly nonobjectivistic and the quasi-objectivistic interpretation. Conclusions are differing largely in different interpretations. We also distinguish between two kinds of locality, viz., macrolocality and Einstein/Bell locality. From a classical model of stochastic measuring processes a definition of Einstein/Bell locality is derived that differs from the Bell/Clauser/Horne/Shimony factorizability condition. It is demonstrated that only in the quasi-objectivistic interpretation the Einstein/Bell locality condition plays a role in the derivation of a Bell inequality for quantities that are experimentally relevant. It is argued that even in this interpretation it is not possible to arrive at the conclusions that the Bell inequalities stem from the locality condition because of the tacit assumption of an additional property, namely the existence of probability distributions conditionalized on the dispersionfree states of the hidden variables. Consideration of a phase space representation of the Schrödinger equation demonstrates that this latter assumption is at odds with the statistics of quantum systems.

Über die Bedeutung der Bellschen Ungleichheiten für das Lokalitätsproblem in unterschiedlichen realistischen Interpretationen der Quantenmechanik

Inhaltsübersicht. Die Beziehung zwischen den Bellschen Ungleichheiten, der Lokalität und der Existenz von gemeinsamen Wahrscheinlichkeitsverteilungen, wird in unterschiedlichen realistischen Interpretationen der Quantenmechanik diskutiert. Wir unterscheiden vier realistische Interpretationen: die objektivistische, die kontextualistische, die enger gefaßte nonobjektivistische und die quasi-objektivistische Interpretation. Die Schlußfolgerungen sind in unterschiedlichen Interpretationen sehr verschieden. Wir machen auch einen Unterschied zwischen zwei Arten der Lokalität, nämlich Makrolokalität und Einstein/Bell-Lokalität. Ausgehend von einem klassischen Modell stochastischer Meßprozesse wird eine Definition der Einstein-Bell-Lokalität hergeleitet, die ungleich der Bell/Clauser/Horne/Shimonyschen Faktorisierungsbedingung ist. Es wird gezeigt, daß nur in der quasi-objektivistischen Interpretation die Einstein/Bell Lokalitätsbedingung eine Rolle beim Herleiten einer Bellschen Ungleichheit für solche Größen, die experimentelle Relevanz haben, spielt. Es wird argumentiert, daß es auch in dieser Interpretation, wegen einer stillschweigenden Annahme einer zusätzlichen Eigenschaft, nämlich die Existenz einer Wahrscheinlichkeitsverteilung, welche konditioniert ist auf dispersionsfreie Zustände der verborgenen Variablen, nicht möglich ist, die Schlußfolgerung zu ziehen, daß die Bellschen Ungleichheiten von der Lokalitätsbedingung abgeleitet werden können. Durch die Betrachtung einer Phasenraumrepräsentation der Schrödingergleichung wird gezeigt, daß diese zusätzliche Annahme im Widerspruch mit der quantenmechanischen Statistik ist.

1. Introduction

Discussions on the significance of the Bell inequalities often lead to confusion because it is not stated explicitly which interpretation is used. In section 2 a number of realist interpretations of quantum mechanics will be discussed. All of these, viz., the objectivistic realist interpretation, the contextualistic one and the nonobjectivistic realist interpretation (both in the strict sense and in its quasi-objectivistic appearance) are based on the existence of so-called hidden variables. They differ, however, in their relation with the quantum mechanical observables. Different interpretations are determined by different roles played by the measuring instruments in the measuring process.

In sect. 3 stochastic processes are discussed in order to obtain an explicit realisation of a measuring process in a realist theory. In section 4 we review the relation between the Bell inequalities (BI), locality, and the existence of a joint probability distribution (jpd) of the four observables that are involved in the BI. The difference between two notions of locality, viz., macrolocality and Einstein/Bell (E/B) locality is stressed. From the results of sect. 3 a definition of E/B locality is derived that differs from the Bell/Clauser/Horne/Shimony (BCHS) factorizability condition [1] in a way that is in agreement with a recent result of Ballentine and Jarrett [2]. On the other hand, it is stressed that in Rastall's derivation [3] of the equivalence of the BI with the existence of a quadrivariate jpd only macrolocality, not E/B locality, is involved.

In section 5 it is investigated for the different realist interpretations of section 2 whether there is any relation between E/B locality and the existence of a jpd (and, hence, the BI). Such a role is found only in one of the interpretations, namely the quasi-objectivistic one. Even in this interpretation, however, it does not seem possible to interpret the BI as a consequence of E/B locality, because of another assumption that is tacitly made, namely that of the existence of probability distributions conditionalized on the dispersionfree states of the hidden variable. In sect. 6 it is demonstrated by means of a discussion of a phase space representation of the free particle Schrödinger equation (Husimi transform) [4], that this latter assumption is at odds with the statistics of quantum systems, and should be regarded as the most probable source of the Bell inequalities.

2. Different Interpretations of Quantum Mechanics

Quantum mechanics deals with probability distributions $p_A(a_i)$ of the (eigen)values a_i of an observable $\mathscr{A}$ represented by an hermitian operator A (for simplicity we consider only discrete spectra). Denoting the state of the system by the density operator ϱ, we have

$$p_A(a_i) = Tr\,\varrho\, M_i, \tag{2.1}$$

in which $\{M_i\}$ is the projection valued measure (or, more generally, the positive operator valued measure [5] corresponding to observable $\mathscr{A}$).

In a phenomenalist/instrumentalist interpretation of quantum mechanics the observable does not refer to a property of the object system. It merely represents a (macroscopic) measurement procedure, and its measurement outcomes a_i are labels of different (macroscopic) phenomena. By the same token the density operator ϱ is not interpreted as a description of the state of the object system, but rather as symbolizing a (macroscopic) measurement procedure. The question of whether the microscopic object system has any properties is left unanswered in this interpretation.

In a realist interpretation of quantum mechanics the microscopic object is assumed to possess properties, often called hidden variables, labelling the dispersionfree states of the object system. The value of this hidden variable, λ say, is thought to determine, either deterministically or statistically, which measurement result a_i will be obtained in a measurement of observable $\mathscr{A}$. Thus, if $p_A(a_i|\lambda)$ is the conditional probability of this result, and if $\varrho(\lambda)$ is the (classical) probability distribution that represents the (result of the) preparation procedure ϱ in terms of the hidden variable λ, then it is assumed that

$$p_A(a_i) = \int_\Lambda d\lambda\, \varrho(\lambda)\, p_A(a_i|\lambda). \tag{2.2}$$

Here Λ is the hidden variable configuration space. In order that the theory can be interpreted as a classical probability theory we should have

$$\varrho(\lambda) \geqq 0, \quad \int_\Lambda d\lambda\, \varrho(\lambda) = 1, \tag{2.3}$$

$$p_A(a_i|\lambda) \geqq 0, \quad \sum_i p_A(a_i|\lambda) = 1. \tag{2.4}$$

In the following we will distinguish between three different varieties of the realist interpretation as defined above, the distinguishing feature being the manner in which the observable $\mathscr{A}$ is interpreted. First, in the objectivistic realist interpretation the observable is thought of as an objective property of the object system, i.e. possessed by the object system independently of the way it is measured. This interpretation is inspired by classical statistical mechanics. Indeed, partitioning Λ into the sets $\Lambda_i = \{\lambda \in \Lambda | A(\lambda) = a_i\}$, we have

$$p_A(a_i) = \int_{\Lambda_i} d\lambda\, \varrho(\lambda) = \int_\Lambda d\lambda\, \varrho(\lambda)\, \chi_{\Lambda_i}(\lambda), \tag{2.5}$$

$\chi_{\Lambda_i}(\lambda)$ being the indicator function of Λ_i. The objectivistic realist interpretation is mentioned here for completeness, because, as is well known, it does not yield a satisfactory interpretation of quantum mechanics.

In the contextualistic realist interpretation the observable is, once more, considered as a property of the object system, be it not an objective one. On the contrary, its value is thought to be liable to change if the measurement context is changed. This interpretation is very much in the spirit of the thought experiments by means of which Bohr and Heisenberg tried to clarify the meaning of Heisenberg's uncertainty relations. In Heisenberg's disturbance theory of measurement [6], for instance, momentum is disturbed in a position measurement. This, more generally, holds for any observable incompatible with the observable that is actually measured.

Because in this interpretation the measuring instrument plays an important role, we have to introduce also a hidden variable μ describing the measuring instrument. Indicating the h.v. space for the measuring instrument of observable $\mathscr{A}$ by M_A, and the initial distribution over this space by $\varrho_A(\mu)$, the probability distribution of measurement results can be written down according to

$$p_A(a_i) = \int_{\Lambda_i} d\lambda \int_{M_A} d\mu\, \mathscr{T}_{\lambda\mu}\, \varrho(\lambda)\, \varrho_A(\mu). \tag{2.6}$$

Here, the operator $\mathscr{T}_{\lambda\mu}$ is representing the interaction of object system and measuring instrument. It transforms the combined system from its initial state to its final state. It is easy to see that (2.6) can be brought into the form (2.2), with

$$p_A(a_i|\lambda) = \int_{M_A} d\mu\, \varrho_A(\mu) \mathscr{T}^*_{\lambda\mu}\, \chi_{\Lambda_i}(\lambda). \tag{2.7}$$

This expression satisfies the requirements (2.4) of a conditional probability if the operator $\mathscr{T}_{\lambda\mu}$ is a stochastic operator [7], that is,

$$\text{on } L^1(\Lambda \times M_A),\ \mathscr{T}_{\lambda\mu} \geqq 0,\ \|\mathscr{T}_{\lambda\mu}\|_1 = 1. \tag{2.8}$$

In the nonobjectivistic realist interpretation the observable is held to be a property of the measuring instrument rather than a property of the object system. Like in the phenomenalist/instrumentalist interpretation the measurement result a_i refers to a macroscopic phenomenon, viz., the position of a macroscopic pointer. The difference with phenomenalism is, that in a realist interpretation the microscopic object is assumed to have a hidden variable λ. Denoting by M_{a_i} the subset of M_A corresponding with the measurement result a_i, we now obtain

$$p_A(a_i) = \int_\Lambda d\lambda \int_{M_{a_i}} d\mu\, \mathscr{T}_{\lambda\mu}\, \varrho(\lambda)\, \varrho_A(\mu)\,. \tag{2.9}$$

This, once again, can be brought into the form (2.2), with

$$p_A(a_i|\lambda) = \int_{M_A} d\mu\, \varrho_A(\mu)\, \mathscr{T}^*_{\lambda\mu} \chi_{M_{a_i}}(\mu)\,, \tag{2.10}$$

$\chi_{M_{a_i}}(\mu)$ being the indicator function of M_{a_i}. As in the contextualistic case this expression (2.10) is a conditional probability if $\mathscr{T}_{\lambda\mu}$ is a stochastic operator.

As in the phenomenalist/instrumentalist interpretation, also in the nonobjectivistic realist interpretation, taken in the strict sense, it does not make sense to attribute meaning to an observable that is not actually measured. In realist theories of the kind we are studying here, however, it is possible to entertain a nonobjectivism of less strict a nature. Although the observable remains a property of the measuring instrument, it yet can be attributed to the object system in the following sense: if the object is in h. v. state λ, it has the property to induce a transition of the measuring instrument from its initial state into a final state a_i as soon as the measuring instrument is applied to the object system. Then, the probability $p_A(a_i|\lambda)$ can be attributed, counterfactually, to the object also if the observable $\mathscr{A}$ is not actually measured. We shall indicate this kind of nonobjectivism as the quasi-objectivistic realist interpretation.

3. Stochastic Processes

In this section we will study stochastic processes described by differential equations of the type

$$\frac{\partial \varrho(\lambda, t)}{\partial t} = T_\lambda \varrho(\lambda, t)\,, \tag{3.1}$$

$$T_\lambda = \sum_{n=1}^{N} \frac{\partial^n}{\partial \lambda^n} a_n(\lambda)\,. \tag{3.2}$$

For simplicity we take $\Lambda = \mathbb{R}^1(-\infty, \infty)$.

If the coefficients $a_n(\lambda)$ are constants, the solution can be given explicitly according to

$$\varrho(\lambda, t) = \frac{1}{2\pi} \int dl\, \tilde{\varrho}(l, 0)\, e^{il\lambda + tT(il)}\,, \tag{3.3}$$

$$\tilde{\varrho}(l, 0) = \int d\lambda\, \varrho(\lambda, 0)\, e^{-il\lambda}\,, \tag{3.4}$$

$$T(il) = \sum_{n=1}^{N} a_n (il)^n\,. \tag{3.5}$$

The Green's function $G(\lambda, \lambda', t)$, being defined as the solution of equation (3.1) with initial condition $\varrho(\lambda, 0) = \delta(\lambda - \lambda')$, is given by

$$G(\lambda, \lambda', t) = \frac{1}{2\pi} \int dl\, e^{il(\lambda - \lambda') + tT(il)}\,. \tag{3.6}$$

Not every differential equation of the type (3.1) describes a classical stochastic process. In order that this be the case $G(\lambda, \lambda', t)$ should be interpretable as a conditional probability distribution. Hence, the integral (3.6) should exist and satisfy (2.3) for any value of λ'.

3.1. Measurement

Let us consider the following equation, describing the interaction between the object system and a measuring instrument for observable $\mathscr{A}$:

$$\frac{\partial \varrho(\lambda, \mu, t)}{\partial t} = -a\lambda \frac{\partial \varrho}{\partial \mu} + T_\lambda \varrho, \tag{3.7}$$

$$\varrho(\lambda, \mu, 0) = \varrho(\lambda, 0)\, \varrho_A(\mu, 0). \tag{3.8}$$

If we assume also M_A to be $R^1(-\infty, \infty)$, $\varrho_A(\mu, 0)$ can be identified with $\varrho_A(\mu)$ in (2.6). In (3.7) a is a constant. For constant coefficients $a_n(\lambda)$ the solution of (3.7) can also be given explicitly by means of its Green's function

$$G_A(\lambda, \lambda', \mu, \mu', t) = \left[\frac{1}{2\pi}\right]^2 \int dl\, dm\, e^{i(l\lambda + m\mu)} e^{-i(l+amt)\lambda' - im\mu' + (am)^{-1} \int_l^{l+amt} T(il')dl'}. \tag{3.9}$$

In the nonobjectivistic interpretation we are interested in the final probability distribution of the measuring instrument

$$\varrho_A(\mu, t) = \int d\lambda\, \varrho(\lambda, \mu, t) = \int d\lambda\, \varrho(\lambda, 0)\, H_A(\mu - \text{at}\, \lambda, t), \tag{3.10}$$

$$H_A(\mu, t) = \frac{1}{2\pi} \int d\mu'\, \varrho_A(\mu', 0) \int dm\, e^{im(\mu - \mu') + (am)^{-1} \int_0^{amt} T(il')dl'}. \tag{3.11}$$

Since (3.10) is a convolution integral it is possible to calculate $\varrho(\lambda, 0)$ if $\varrho_A(\mu, t)$ is known. Indeed, for the Fourier transforms of $\varrho(\lambda, 0)$, $\varrho_A(\mu, t)$ and $H(\mu, t)$ we find the relation

$$\tilde{\varrho}(l, 0) = \tilde{\varrho}_A(l/at, t)/\tilde{H}_A\,(l/at, t). \tag{3.12}$$

This result justifies the interpretation of (3.7) as describing a measuring process. It is also interesting to note that this measuring process is nondisturbing. Indeed, by integrating (3.7) over μ it can easily be verified that $\int d\mu\, \varrho(\lambda, \mu, t)$ satisfies equation (3.1).

As an example, let us consider equation (3.7) for the ordinary diffusion process,

$$\frac{\partial \varrho}{\partial t} = -a\lambda \frac{\partial \varrho}{\partial \mu} + D \frac{\partial^2 \varrho}{\partial \lambda^2}. \tag{3.13}$$

This equation is a so-called linear Fokker-Planck equation [8] which can also be solved explicitly. Its Green's function is given by

$$G_A(\lambda, \lambda', \mu, \mu', t) = \frac{\sqrt{3}}{2\pi a D t^2} \times \exp\left\{\frac{-3(\mu - \mu')^2 + 3at(\mu - \mu')(\lambda + \lambda') - a^2t^2(\lambda^2 + \lambda'^2 + \lambda\lambda')}{Da^2t^3}\right\}. \tag{3.14}$$

From this we find for (3.11)

$$H_A(\mu, t) = \int d\mu'\, \varrho_A(\mu', 0) \left[\frac{3}{4\pi\, Da^2t^3}\right]^{1/2} \exp \frac{-3}{4Da^2t^3} (\mu - \mu')^2. \tag{3.15}$$

It is evident from (3.10) and (3.15) that for equation (3.13) it is possible to define a conditional probability satisfying (2.4). Indeed the conditional probability $p_{tA}(\mu|\lambda)$ of finding, at t, measurement result μ if the object was in λ at $t = 0$, can be defined according to

$$p_{tA}(\mu|\lambda) := H_A(\mu - at\lambda, t). \tag{3.16}$$

3.2. EPR-like Experiments

Let two objects, having hidden variables λ_1 and λ_2, respectively, both satisfy the same evolution equation

$$\frac{\partial \varrho_i(\lambda_i, t)}{\partial t} = T_{\lambda_i} \varrho_i(\lambda_i, t), \quad i = 1, 2. \tag{3.17}$$

The two particles are supposed not to interact. They are prepared at $t = 0$ in a correlated state $\varrho(\lambda_1, \lambda_2, 0)$. An EPR-like experiment, in which the particles are allowed to separate and, subsequently, observables $\mathscr{A}_1$ and $\mathscr{A}_2$ of particles 1 and 2, respectively, are measured jointly, can be simulated by means of the equation

$$\frac{\partial \varrho}{\partial t} = -a_1\lambda_1 \frac{\partial \varrho}{\partial \mu_{A_1}} - a_2\lambda_2 \frac{\partial \varrho}{\partial \mu_{A_2}} + [T_{\lambda_1} + T_{\lambda_2}]\, \varrho. \tag{3.18}$$

It is easily demonstrated that the Green's function for this equation is just the product of the Green's functions for the two particles separately, each described by (3.7). Thus,

$$\begin{aligned} &G_{A_1A_2}(\lambda_1, \lambda_1', \lambda_2, \lambda_2', \mu_{A_1}, \mu'_{A_1}, \mu_{A_2}, \mu'_{A_2}, t) \\ &= G_{A_1}(\lambda_1, \lambda_1', \mu_{A_1}, \mu'_{A_1}, t)\, G_{A_2}(\lambda_2, \lambda_2', \mu_{A_2}, \mu'_{A_2}, t). \end{aligned} \tag{3.19}$$

If the initial condition is given by

$$\varrho(\lambda_1, \lambda_2, \mu_{A_1}, \mu_{A_2}, 0) = \varrho(\lambda_1, \lambda_2, 0)\, \varrho_{A_1}(\mu_{A_1}, 0)\, \varrho_{A_2}(\mu_{A_2}, 0),$$

the probability $\varrho_{A_1A_2}(\mu_{A_1}, \mu_{A_2}, t)$ can be given, analogously to (3.10), as

$$\begin{aligned} &\varrho_{A_1A_2}(\mu_{A_1}, \mu_{A_2}, t) = \int d\lambda_1\, d\lambda_2\, \varrho(\lambda_1, \lambda_2, 0)\, p_{tA_1}(\mu_{A_1}|\lambda_1)\, p_{tA_2}(\mu_{A_2}|\lambda_2), \\ &p_{tA_i}(\mu_{A_i}|\lambda_i) = \int d\lambda_i' \int d\mu'_{A_i}\, \varrho_{A_i}(\mu'_{A_i})\, G_{A_i}(\lambda_i, \lambda_i', \mu_{A_i}, \mu'_{A_i}, t), \quad i = 1, 2. \end{aligned} \tag{3.20}$$

4. Bell Inequalities, Locality, and Joint Probability Distributions

The Bell inequalities were derived [9] in order to investigate the (im)possibility of realist interpretations as discussed in sect. 2. They refer to EPR-like experiments of the kind described in sect. 3, in which $\mathscr{A}_1$ can be chosen from a set of two incompatible observables $\mathscr{A}$ and $\mathscr{A}'$, say, and $\mathscr{A}_2$, analogously from the incompatible pair $\mathscr{B}, \mathscr{B}'$. The $\mathscr{A}_1$-observables are compatible with the $\mathscr{A}_2$-observables.

In deriving his inequalities Bell did consider only h. v. theories satisfying a locality property, which he deemed essential for the derivation. His conlusion was (also [1]) that only h. v. theories not satisfying this condition, so-called nonlocal h. v. theories, remain as good candidates for a realist interpretation of quantum mechanics. The relevance of this locality condition is the subject of the present section.

4.1. Macrolocality and Einstein-Bell Locality

In order to avoid confusion we should first distinguish between two different notions of locality, viz. macrolocality and Einstein/Bell locality. Macrolocality is expressed in terms of the probability distributions of the measurement results. It says that in the EPR-like measurements of $\mathscr{A}$ and $\mathscr{B}$ or $\mathscr{A}$ and $\mathscr{B}'$, respectively, the probability of the measurement results of $\mathscr{A}$ is independent of the choice $\mathscr{B}$ or $\mathscr{B}'$. Thus, in an evident notation,

$$\sum_k p_{AB}(a_i, b_k) = \sum_l p_{AB'}(a_i, b_l) = p_A(a_i). \tag{4.1}$$

Quantum mechanics satisfies macrolocality, which is warranted by the compatibility of the $\mathscr{A}_1$- and $\mathscr{A}_2$-observables. For this reason this cannot be the kind of locality presupposed by Bell. Indeed, Bell requires from his hidden variables a stronger form of locality (for this reason called strong locality in [2]), to the effect that the independence of the measurement results is not only statistically true, but holds for **each individual measurement result** a_i. Since this coincides with the locality notion as deployed by Einstein, we refer to it as Einstein/Bell locality. It is clear that E/B locality implies macrolocality, but not vice versa.

By Clauser and Shimony [1] a statistical version of the E/B locality condition is introduced, viz.

$$\begin{aligned} P_{AB}(a_i, b_k) &= \int d\lambda\, \varrho(\lambda)\, p_{AB}(a_i, b_k \mid \lambda), \\ P_{AB}(a_i, b_k \mid \lambda) &= p_A(a_i \mid \lambda)\, p_B(b_k \mid \lambda). \end{aligned} \tag{4.2}$$

Equation (4.2) is generally referred to as the BCHS (Bell, Clauser, Horne, Shimony) locality condition. The analysis by Ballentine and Jarrett [2] demonstrates that, strictly speaking, the BCHS-condition is not a locality condition but a condition of, what they call, predictive completeness, However, this predictive completeness is implied by E/B (strong) locality. Indeed, as can be seen from (3.20) by identifying λ with the pair (λ_1, λ_2), the EPR-measurements considered there, being E/B local by construction, satisfy a separability criterion which is a special case of the BCHS-condition. In order to avoid confusion, in the following the E/B locality condition will be represented by

$$p_{AB}(a_i, b_k) = \int d\lambda_1\, d\lambda_2\, \varrho(\lambda_1, \lambda_2)\, p_{AB}(a_i, b_k \mid \lambda_1, \lambda_2), \tag{4.3}$$

$$p_{AB}(a_i, b_k \mid \lambda_1, \lambda_2) = p_A(a_i \mid \lambda_1)\, p_B(b_k \mid \lambda_2). \tag{4.4}$$

4.2. Locality and the Bell Inequalities

It is a widespread belief that a) the Bell inequalities are essentially a consequence of E/B locality, and that b) since quantum mechanics does not satisfy the BI, the quantum world, although macrolocal, is not E/B local. It should be noted, however, that a) would be true only if no other assumptions than locality were made, thus avoiding the possibility to blame the BI on these other assumptions instead of on the locality assumption. As a matter of fact, such an additional assumption is necessary in order to be able to draw conclusion b). Indeed, this conclusion hinges on the mere **possibility** of a realist interpretation of quantum mechanics. Without such an assumption quantum mechanics can be interpreted, in a completely satisfactory way, as describing a (macro)local world. Only for (certain) **realist** theories universal validity of the BI is derivable. For this reason realism itself, be it either local or nonlocal realism, might be the origin of the Bell inequalities.

In his original derivation Bell started from the idea that only **local** realism would be liable to refutation because he thought that Bohm's 1952 theory [10] provided evidence of the existence of a **nonlocal** h. v. theory equivalent with quantum mechanics. In our opinion, however, it is questionable whether Bohm's theory can be interpreted as a nonlocal h. v. theory [11]. For this reason Bell's point of departure may be too restrictive and nonlocal realism might be refutable as well. In the following this will be investigated for the realist interpretations introduced in section 2.

By Rastall [3] (also Fine [12]) the following theorem is derived in a very lucid way: If the four EPR probabilities $p_{AB}(a_i, b_k)$, $p_{AB'}(a_i, b_l')$, $p_{A'B}(a_j', b_k)$ and $p_{A'B'}(a_j', b_l')$ satisfy macrolocality (in the sense of (4.1)), then the Bell inequalities are necessary and sufficient for the existence of a quadrivariate joint probability distribution $p(a_i, a_j', b_k, b_l')$ having the EPR probabilities as marginals (In Rastall's proof the observables $\mathscr{A}$, $\mathscr{A}'$, $\mathscr{B}$, and $\mathscr{B}'$ are restricted to dichotomic observables having values ± 1).

It is regrettable that the role played by the locality issue in Rastall's derivation is confused by the question of whether a derivation of the BI is possible without the introduction of hidden variables [13]. Restricting ourselves to realist theories as discussed in section 2, in any case it is clear that in Rastall's derivation no E/B locality is presupposed, i.e. the EPR probabilities need not satisfy (4.4). Notice that compatibility of E/B nonlocality and macrolocality, although rather counterintuitive, is not logically impossible. Indeed, the conclusion that the Aspect experiments [15] prove the nonlocality of the quantum world can not be drawn without the assumption of such a compatibility.

Since macrolocality is not at issue in the Bell/EPR problem, we may conclude from Rastall's derivation that the existence of the jpd $p(a_i, a_j', b_k, b_l')$ is the real cause of the BI. The only way E/B locality could interfere with this conclusion would be if it could be proven that the existence of the jpd is implied by E/B locality. This will be investigated in the next section for the different interpretations introduced in sect. 2.

5. Joint Probability Distributions in Different Interpretations of Quantum Mechanics

5.1. The Phenomenalist-instrumentalist Interpretation

For completeness we include the phenomenalist/instrumentalist interpretation in our discussion. Since in $p(a_i, a', b_k, b_l')$ the values of a_i, a_j', b_k, and b_l' refer to phenomena that should be combinable into one single event, in this interpretation a jpd only makes sense if $\mathscr{A}$, $\mathscr{A}'$, $\mathscr{B}$, and $\mathscr{B}'$ are jointly measurable. In orthodox interpretations of quantum mechanics this is held to be impossible because of incompatibility of e.g. $\mathscr{A}$ and $\mathscr{A}'$. Hence, the BI cannot be derived in such interpretations. If the possibility of the joint measurement of incompatible observables is not excluded in principle ([16–18]) the conclusion should be that the experimental results of the joint measurement of $\mathscr{A}$, $\mathscr{A}'$, $\mathscr{B}$, and $\mathscr{B}'$ should satisfy the BI. This will be discussed in more detail in a future publication.

5.2. The Objectivistic Realist Interpretation

As far as realist interpretations are concerned we should make a distinction between the three varieties introduced before. From these the objectivistic realist interpretation is dealt with in the easiest manner. Here, the values a_i, a_j', b_k, and b_l' are attributed simultaneously to the object as properties of the latter, analogously to classical mechanical quantities, i.e. independent of the measurement situation. This guarantees the existence of a jpd. Hence, this interpretation is ruled out by the BI. This fact does not depend on whether the interaction between the two particles in the EPR experiment is E/B local or nonlocal.

5.3. The Contextualistic Realist Interpretation

In the contextualistic realist interpretation a reasoning exists (Dieks [19]) purporting to exclude only E/B local theories. The reasoning goes like this:

a) For E/B local theories, satisfying (4.4), it is possible to construct a jpd according to

$$\begin{aligned} &p(a_i, a'_j, b_k, b'_l) \\ &= \int d\lambda_1\, d\lambda_2\, \varrho(\lambda_1, \lambda_2)\, p_A(a_i \mid \lambda_1)\, p_{A'}(a'_j \mid \lambda_1)\, p_B(b_k \mid \lambda_2)\, p_{B'}(b'_l \mid \lambda_2). \end{aligned} \tag{5.1}$$

This jpd has the four EPR probabilities (4.3) as its marginals, satisfying (4.4). Hence, E/B local theories are ruled out by the BI.

b) In case of E/B nonlocality (4.4) is not satisfied. For this reason the jpd (5.1) cannot be constructed, and the BI do not seem to be derivable.

From this reasoning, however, it cannot be concluded that E/B nonlocal theories are possible. This is so because E/B locality, although sufficient, is not necessary for the existence of a jpd. It is not ruled out that a jpd, different from (5.1), can be constructed in case of E/B nonlocality. As a matter of fact, as demonstrated by Rastall [13], this is possible (under the assumption of macrolocality) if and only if the BI are satisfied. Also the contextualistic realist interpretation would be ruled out, both in the E/B local and nonlocal case, by the existence of a jpd.

In the above reasoning the nonexistence of a jpd hinges on the nonlocal influence of one measurement on the outcome of a distant measurement performed jointly. In a contextualistic realist interpretation of quantum mechanics, however, we are led to the nonexistence of jpd's also for E/B local theories, if we take into account the influence of the nearby measuring instrument on the properties of the object, rather than the influence of the far one. As a matter of fact, this latter idea is at the very basis of quantum theory as entrenched by Heisenberg's disturbance theory of measurement [6]. As is well known, in this theory, if $\mathscr{A}'$ is measured, then the incompatible quantity $\mathscr{A}$ is disturbed by the interaction with the measuring instrument. This disturbance is a local affair, summoned precisely to account for the nonexistence of jpd's of two incompatible observables (like position and momentum). In Heisenberg's theory observables that are compatible with the one that is actually measured are left undisturbed, thus allowing for macrolocality. Hence, on the basis of Heisenberg's disturbance theory of measurement we have

$$p_{A'}(a_i) \neq p_A(a_i), \tag{5.2}$$

$p_{A'}(a_i)$ denoting the distribution of property $\mathscr{A}$ if $\mathscr{A}'$ is measured. Since any jpd of $\mathscr{A}$, $\mathscr{A}'$, $\mathscr{B}$, and $\mathscr{B}'$ would imply a unique marginal $p(a_i)$, (5.2) is inconsistent with the existence of one jpd for all EPR experiments. For this reason the jpd (5.1) is unphysical in the contextualistic interpretation, and any consequences following from it — like the BI — are lacking physical relevance.

In the contextualistic realist interpretation we have a different jpd of $\mathscr{A}$, $\mathscr{A}'$, $\mathscr{B}$, and $\mathscr{B}'$ for each of the four possible EPR measurement situations. Thus, in a local theory we have

$\mathscr{A}\,\mathscr{B}$ measurement:

$$p_{AB}(a_i, a'_j, b_k, b'_l) = \int d\lambda_1\, d\lambda_2\, \varrho(\lambda_1, \lambda_2)\, p_A(a_i, a'_j \mid \lambda_1)\, p_B(b_k, b'_l \mid \lambda_2), \tag{5.3}$$

$\mathscr{A}'\,\mathscr{B}$ measurement:

$$p_{A'B}(a_i, a'_j, b_k, b'_l) = \int d\lambda_1\, d\lambda_2\, \varrho(\lambda_1, \lambda_2)\, p_{A'}(a_i, a'_j \mid \lambda_1)\, p_B(b_k, b'_l \mid \lambda_2). \tag{5.4}$$

Comparing (5.3) and (5.4) it can be seen that (5.2) is a consequence of the inequality of $p_A(a_i, a'_j \mid \lambda_1)$ and $p_{A'}(a_i, a'_j \mid \lambda_1)$, which is a purely local affair.

Of course, it is possible to derive a BI from each of the jpd's like (5.3) and (5.4). In the contextualistic realist interpretation, however, EPR-measurement outcomes need not satisfy the BI because the experimental EPR probabilities are derived from different jpd's. In the inequality

$$|\langle AB\rangle + \langle AB'\rangle + \langle A'B\rangle - \langle A'B'\rangle| \leqq 2,$$

obtained for an $\mathscr{A}\mathscr{B}$-measurement from (5.3) (for dichotomic observables having values ± 1), only $\langle AB\rangle$ has experimental relevance, $\langle A'B\rangle$ being different from the expectation value obtained in an $\mathscr{A}'\mathscr{B}$ measurement because of (5.2). For this reason the jpd (5.3) is not excluded on account of the violation of the BI by quantum mechanics.

From the reasoning given above it follows that the contextualistic realist interpretation of quantum mechanics is not ruled out by the BI. There is, however, another reason why this interpretation is not very attractive. If we consider the role played by the measuring instrument, we see that it only acts as a source of disturbance for the object system. The measurement outcome being attributed to the final state of the object, rather than to the measuring instrument, the latter fails to bear out its intended task of amplifying microscopic information to the macroscopic domain of direct observation. In this interpretation information regarding the measuring instruments is not considered as relevant. If this would be possible, however, why then not leave out the measuring instrument completely?

5.4. The Nonobjectivistic Realist Interpretation

In the nonobjectivistic realist interpretation the role of the measuring instrument is taken more seriously. For this reason this interpretation seems to be the more appropriate one. Taken in the strict sense (cf. sect. 2) the nonobjectivistic realist interpretation is rather analogous to the phenomenalist/instrumentalist one, also with respect to the issue of the BI, the latter applying only to the joint measurement of $\mathscr{A}$, $\mathscr{A}'$, $\mathscr{B}$, and $\mathscr{B}'$.

5.5. The Quasi-objectivistic Realist Interpretation

More interesting for our present discussion is the quasi-objectivistic version of nonobjectivistic realism. Applying this interpretation to the Bell/EPR-problem, we have to attribute the four conditional probabilities $p_A(a_i|\lambda)$, $p_{A'}(a'_j|\lambda)$, $p_B(b'_k|\lambda)$, and $p_{B'}(b'_l|\lambda)$ to the object, independently of whether the observable is actually measured or not. This implies the existence of the jpd

$$p(a_i, a'_j, b_k, b'_l) = \int d\lambda\, \varrho(\lambda)\, p_A(a_i|\lambda)\, p_{A'}(a'_j|\lambda)\, p_B(b_k|\lambda)\, p_{B'}(b'_l|\lambda), \tag{5.5}$$

which would be relevant to the experimental situation in which it is possible to prepare the object in a dispersionless state λ in a repeatable way, and measure, on each preparation, one of the observables $\mathscr{A}$, $\mathscr{A}'$, $\mathscr{B}$, or $\mathscr{B}'$ at a time. Then the jpd (5.5) can be obtained by repeating the experiment for the different observables and different values of λ, taking care that for each observable the distribution $\varrho(\lambda)$ is the same. This procedure is completely independent of the issue of locality since each observable is measured singly. Since the measurement procedure given above differs from the usual way in which EPR/Bell experiments are carried out, we need not be worried that the BI should be satisfied on behalf of the existence of the jpd (5.5). Actually, as stressed by de Baere ([20, 21]), in real experiments we do not have sufficient control of the object system to be able to know the dispersionless state it is prepared in. For this reason we are not able to reproduce the same λ in a different experiment, thus lacking a necessary prerequisite for the comparison of the jpd (5.5) with experimental correlations obtained in EPR-like experiments.

In case of E/B locality (5.5) reduces to (5.1). From (2.4), (4.3) and (4.4) it then follows that all probabilities $p_{AB}(a_i, b_k)$, etc., of EPR/Bell experiments are marginals of the same jpd (5.1). It would seem, then, that in the quasi-objectivistic interpretation these experiments should satisfy the BI because of E/B locality. As discussed already in sect. 4, however, we should be aware of the fact that, apart from locality, we also assumed the existence of conditional probabilities like $p_A(a_i|\lambda)$, etc., satisfying (2.4), thus presupposing the possibility of preparing the object in a dispersionless state λ before measuring observable $\mathscr{A}$. As can be seen from the example discussed in the next section, it is in the spirit of quantum mechanics to doubt this latter presupposition, quite independently of the (non)locality question. The nonexistence of the jpd (5.1) or (5.5) could be due to the impossibility, within the domain of application of quantum mechanics, to prepare the object system in a dispersionless hidden variable state λ.

6. On the Possibility of a Classical Description of Quantum Systems

In this section we will discuss the Husimi transform [4] of the free particle Schrödinger equation, with the intention to perform a critical examination of the existence of conditional probabilities like $p_A(a_i|\lambda)$, that play such a prominent role in the quasi-objectivistic realist interpretation.

Defining a coherent state of a one-dimensional system according to

$$\varphi^{(s)}_{q,p}(x) = (\pi\hbar s)^{-1/4} \exp\left[-\frac{(x-q)^2}{2\hbar s^2} + \frac{ixp}{\hbar}\right], \tag{6.1}$$

the Husimi transform of the density operator $\varrho(t)$ is defined as

$$\varrho(q, p, t) := \frac{1}{h} \langle\varphi^{(s)}_{q,p}|\,\varrho(t)\,|\varphi^{(s)}_{q,p}\rangle. \tag{6.2}$$

From the free particle Schrödinger equation we obtain for (6.2) the evolution equation

$$\frac{\partial\varrho(q, p, t)}{\partial t} = \left[-\frac{p}{m}\frac{\partial}{\partial q} - \frac{\hbar}{2s^2 m}\frac{\partial^2}{\partial q\,\partial p}\right]\varrho(q, p, t), \tag{6.3}$$

which is a phase space representation of the Schrödinger equation ("antinormal ordering rule" [22]). Because the states (6.1) constitute an overcomplete set of states ([22]), it is clear from the definition (6.2) that, with $\lambda = (q, p)$, $\varrho(q, p, t)$ satisfies (2.3), for any initial density operator ϱ, for all later values of t.

The solution to (6.3) can be obtained, analogously to (3.9), in the form

$$\varrho(q, p, t) = \int dq'\, dp'\, \varrho(q', p', 0)\, G(q, q', p, p', t), \tag{6.4}$$

$$G(q, q', p, p', t) = \frac{1}{(2\pi)^2}\int dk \int dl\, e^{ik(q-q')+il(p-p')} e^{-i\frac{ktp}{m}}\, e^{\frac{\hbar kt}{2s^2m}\left[-\frac{kt}{2m}+l\right]}. \tag{6.5}$$

For two noninteracting free particles the formalism can be extended according to

$$\frac{\partial\varrho}{\partial t} = \left[-\frac{p_1}{m}\frac{\partial}{\partial q_1} - \frac{\hbar}{2s^2m}\frac{\partial^2}{\partial q_1\partial p_1} - \frac{p_2}{m}\frac{\partial}{\partial q_2} - \frac{\hbar}{2s^2m}\frac{\partial^2}{\partial q_2\,\partial p_2}\right]\varrho,$$
$$\varrho = \varrho(q_1, p_1, q_2, p_2, t), \tag{6.6}$$

with solution

$$\varrho(q_1, p_1, q_2, p_2, t)$$
$$= \int dq_1'\, dp_1'\, dq_2'\, dp_2'\, \varrho(q_1', p_1', q_2', p_2', 0)\, G(q_1, q_1', p_1, p_1', t)\, G(q_2, q_2', p_2, p_2', t). \tag{6.7}$$

It is interesting to note that this solution of a fully quantum mechanical problem satisfies E/B locality. This conclusion remains valid also if the particles have interacted during $t < 0$.

In order to investigate the nonclassical behaviour that is peculiar to phase space representations of quantum mechanics, we will now consider the Green's function (6.5) somewhat more closely. If (6.4) is to be interpreted analogously to (2.2), the Green's function $G(q, q', p, p', t)$ should be interpretable as a conditional probability $p(q, p, t \mid q', p')$ of finding the object at t in (q, p) if it started at $t = 0$ in (q', p'). This implies that for any (q', p') the Green's function $G(q, q', p, p', t)$ should be a nonnegative and integrable function of (q, p). This, however, is not the case. As follows from (6.2) and (6.4), $G(q, q', p, p', t)$ can be interpreted as a continuous linear functional (distribution) on the space of test functions generated by the "good" functions $\langle \varphi_{q,p}^{(s)} | \varrho | \varphi_{q,p}^{(s)} \rangle$, ϱ a density operator. This space, however, does not contain the dispersionfree states $\varrho(q, p, 0) = \delta(q - q')\, \delta(p - p')$ as can be seen by taking

$$\varrho(q, p, 0) = (2\pi\sigma_q\sigma_p)^{-1} \exp\left[-\frac{q^2}{2\sigma_q^2} - \frac{p^2}{2\sigma_p^2}\right].$$

For this initial condition the solution (6.4) turns out to diverge for $t > 2s^2m\, \sigma_q\sigma_p[\hbar(\hbar - 2s^2\sigma_p^2)]^{-1/2}$. Hence, for $\sigma_p^2 < \hbar(2s^2)^{-1}$ the solution diverges if t is large enough.

Mathematically speaking this result is not unexpected, since the equation (6.3) is not a Fokker-Planck equation in the strict sense. As a matter of fact, the operator $\mathcal{T}_{q,p} = \exp Tt$, $T = -\dfrac{p}{m}\dfrac{\partial}{\partial q} - \dfrac{\hbar}{2s^2m}\dfrac{\partial^2}{\partial q\, \partial p}$, is not a stochastic operator satisfying (2.8). This comes about because the operator $\dfrac{\partial^2}{\partial q\, \partial p}$ is not a positive definite operator on $L^1(\mathrm{R}^1 \times \mathrm{R}^1)$ (cf. van Kampen [8]).

Evidently, the stochastics of this phase space representation of quantum mechanics does not resemble the stochastics of classical mechanics, the difference being crucially represented by the impossibility to attribute, within the domain of quantum mechanics, a physical meaning to dispersionfree states. This is at the heart of the difficulties encountered by realist interpretations of quantum mechanics. The problem has its roots already in the physics of a single particle and is therefore independent of the (non)locality question. As seen from (6.7) correlations between noninteracting particles can be treated in a classical way (apart from the nonclassical behaviour of each particle separately). It does not make sense to treat the particle pair classically because it does not make sense to treat each of the single particles classically.

7. Conclusions

We investigated the relation between the Bell inequalities, locality, and the existence of a joint probability distribution of four observables, in different realist interpretations of quantum mechanics. Conclusions are strongly dependent on the interpretation that is chosen. Thus, the objectivistic realist interpretation is ruled out by the applicability of the BI to EPR-like experiments. On the other hand, the contextualistic realist interpretation, also on the assumption of E/B locality, is not affected by the BI. It is an acceptable interpretation from this latter point of view, even though it is less attractive for other reasons.

The nonobjectivistic realist interpretation, taken in the strict sense, can be compared with the phenomenalist/instrumentalist one, since in both interpretations the BI are inapplicable to EPR-like experiments. Because of the comparative weakness of its realist assumptions this kind of interpretation may be felt to be not completely satisfactory to the confirmed realist, who, for this reason, may prefer the quasi-objectivistic realist interpretation. In this latter interpretation the problem of the BI is particularly stubborn. However, in recompense it yields the opportunity to gain a deeper insight into the relative insignificance of the issue of locality compared with the peculiarities of the quantum statistics of a single particle. This result bears out the intuition that the problem of quantum mechanics is with the existence of incompatible quantities, and, hence, as already remarked by Bub [23] the addition to the incompatible pair $\mathscr{A}$, $\mathscr{A}'$ of observables $\mathscr{B}$ and $\mathscr{B}'$ that are compatible with the former ones, can only confuse the issue. It seems to us that this confusion has materialized in the form of the locality problem.

Of course, our result is rather negative in the sense that it repudiates nonlocality as the source of quantum correlations. It does not provide for a solution of the problem that is different from a reference to the time-honoured argument that a quantum state cannot be viewed upon as a classical ensemble of dispersionless states of the hidden variable. It remains completely unclear whether it is possible at all to provide quantum statistics with a realist basis, and, if so, in what sense the models discussed in sect. 2 should be modified in order to arrive at an acceptable one. Yet, we hope that the present exposition will contribute to an increase of the interest for the representation of quantum fluctuations in terms of realist theories, at the cost of all the effort that is made in chasing a nonlocality that is theoretically illusive and for which there exists not a bit of experimental evidence.

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