QUANTUM NONLOCALITY AND BELL'S INEQUALITIES

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ABSTRACT

A number of additional assumptions is discussed playing a role in derivations of the Bell inequalities along with the locality condition. An analogy between quantum mechanics and statistical thermodynamics is exploited to demonstrate that the existence of conditional probabilities $p(a_i|\lambda)$, generally assumed in stochastic hidden variables theories, may be questionable within the domain of application of quantum mechanics. It is demonstrated how this could block derivations of the Bell inequalities and of the Kochen-Specker theorem. The analogy also provides a possible understanding of the nonlocality of quantum mechanics as evidenced by the Borchers-Schlieder theorem.

1. INTRODUCTION

It is widely believed that experimental violation of the Bell inequalities (BI) is a consequence of the nonlocality of the reality underlying quantum mechanics. It is remarkable that this belief is equally shared by adherents of the orthodox (Copenhagen) interpretation of quantum mechanics and by adversaries of this latter interpretation, but because of completely different reasons. Whereas the BI are derived from a (local) hidden variables theory, thus assuming the *incompleteness* of quantum mechanics, an orthodox reading of the problem raised by Einstein, Podolsky and Rosen (EPR) [1] on the contrary derives nonlocality from the *completeness* of quantum mechanics.

This remarkable agreement suggests that something may be wrong with the logic employed in the derivations, since $a \to b$ and $\sim a \to b$ would mean that both a and $\sim a$ are implied by $\sim b$, thus making $\sim b$, i.e., locality, *logically* impossible. Indeed, Einstein [2] may have been somewhat too hasty in his conclusion from the implication

$$\text{completeness} \to \text{nonlocality}, \quad (1)$$

that the assumption of *in*completeness of quantum mechanics would solve the locality problem. As a matter of fact, nonlocality is a problem *also* in an ensemble-interpretation of quantum mechanics, if the state function is interpreted in a realist sense [3]. Evidently, apart from the completeness issue there are more factors involved that may play a role in the derivations.

It is important to note that the conclusion of nonlocality, drawn from the violation of BI, is compulsory only if there are no additional assumptions, beside locality, that can be held responsible for the fact that BI can be derived. In section 3 we discuss a number of such additional assumptions. Counterfactual definiteness (CFD) is such an assumption, nowadays generally held to be incompatible with quantum mechanics. We briefly discuss the possibility that the violation of CFD might be a consequence of the assumption of locality, a more extensive discussion, in which this violation is traced back to the problem of joint measurement of incompatible observables, being relegated to another paper [3] (see also [4]). Second, the careful discussion by Bell of the issue of (in)completeness in his theory of local beables is reviewed, illustrating the role played by the additional assumption of completeness, and corroborating the implication (1) given above. In the third place we pay attention to the question whether the existence of the conditional probabilities $p(a_i|\lambda)$, generally assumed in stochastic hidden variables theories, should be taken as an additional assumption. Our conclusion will be that it should, and that relaxation of the assumption is sufficient to block derivation of the Bell inequalities.

In section 4 we discuss an analogy between quantum mechanics and thermodynamics in order to develop a certain heuristics explaining the irrelevance of the conditional probabilities $p(a_i|\lambda)$ within the domain of application of quantum mechanics. It is also indicated how this could block the Kochen-Specker impossibility proof of hidden variables.

Finally, in section 5 we compare the alleged nonlocality of quantum mechanics following from the violation of the BI with the nonlocality represented by the superluminal propagation of solutions of quantum mechanical propagation equations. The thermodynamic analogy is employed to interpret this latter effect as a consequence of the approximate validity of these equations, i.e., as an artifice of the theory rather than as a feature of reality.

2. BELL'S THEOREM

Derivations of Bell's theorem abound in literature. So we can be very brief and restrict ourselves to a precise description of the preconditions and assumptions involved. As is well known, the Bell theorem concerns so–called Einstein-Podolsky-Rosen experiments, in which a two-particle system (particles 1 and 2) is prepared in an entangled state, and a joint measurement is performed of a pair of (compatible) observables, one for each particle[1]. The measurements are performed after the particles have separated, –the entanglement being conserved–, thus making it possible to consider the two measurements as *local* measurements performed in different regions of space.

[1]It must be noted that it is generally ignored that the Bell proposal is fundamentally different from the original EPR one in the sense that it is essential to the EPR reasoning that a measurement is performed on one particle only, the other particle *not* interacting with any measuring instrument. Nevertheless we shall stick to common usage and refer to the experiments of the Bell type as EPR experiments.

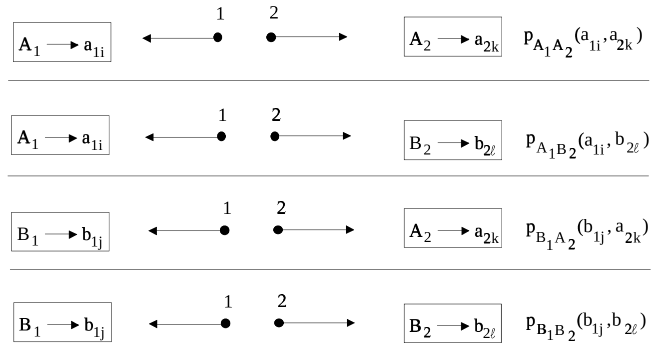

Figure 1: Four EPR experiments involved in the Bell inequality

If A_1 and A_2 are the two pertinent observables, having eigenvalues a_{1i} and a_{2k}, respectively, then the joint measurement probability $p_{A_1A_2}(a_{1i}, a_{2k})$ fully describes the information obtained by the measurement.

Bell's theorem assumes that we can perform four different experiments by choosing the observable measured on particle i, $i = 1, 2$, from a set of two incompatible observables $\{A_i, B_i\}$. Hence, four observables are involved in the theorem, viz., A_1, B_1, A_2, B_2, observables with different indices being compatible, observables with identical indices taken to be incompatible, i.e.,

$$\begin{array}{c} [A_1, B_1]_- \neq 0,\ [A_2, B_2]_- \neq 0, \\ [A_1, A_2]_- = [A_1, B_2]_- = [B_1, A_2]_- = [B_1, B_2]_- = 0. \end{array}$$

The four measurements with their joint measurement probabilities are symbolically represented in figure 1. Bell's theorem now may be formulated, in the form first given by Clauser and Horne [5], as a derivation of the inequality

$$\begin{array}{rcl} 0 & \leq & p_{A_1A_2}(a_{1i}, a_{2k}) - p_{A_1B_2}(a_{1i}, b_{2\ell}) + p_{B_1A_2}(b_{1j}, a_{2k}) + \\ & & + p_{B_1B_2}(b_{1j}, b_{2\ell}) - p_{B_1}(b_{1j}) - p_{A_2}(a_{2k}) \leq 1, \end{array} \tag{2}$$

$$p_{B_1}(b_{1j}) = \sum_\ell p_{B_1B_2}(b_{1j}, b_{2\ell}), \text{ etc.,}$$

from which the inequalities involving correlation functions,

$$|\langle A_1A_2\rangle + \langle A_1B_2\rangle + \langle B_1A_2\rangle - \langle B_1A_2\rangle| \leq 2$$

as originally found by Bell [6], can be easily derived [7] for dichotomic observables having eigenvalues 1 and -1.

In the derivations [5] and [6] of the Bell inequalities a locality condition was presupposed to the effect that an individual measurement result of observable A_1 or B_1 should be independent of whether A_2 or B_2 is measured on particle 2. We shall express this locality requirement by the equality

$$a_{1i}(A_1, A_2) = a_{1i}(A_1, B_2). \tag{3}$$

For the EPR measurements considered here this, indeed, seems to be a reasonable requirement if nonlocal influences of the distant measuring instrument are to be excluded. Note that the equality (3) is stronger than the *statistical* locality requirement

$$p_{A_1}(a_{1i}) = p_{A_1 A_2}(a_{1i}) = p_{A_1 B_2}(a_{1i}) \tag{4}$$

already taken into account in (2). Whereas the equality (4) can be proven to hold rigorously in quantum mechanics, the assumption (3) is transcendent to this theory. It, however, would require a rather strange conspiracy of a nonlocal quantum reality violating (3), to be able to reproduce in an ensemble of measurements the relative frequencies satisfying the statistical locality condition (4). This would mean that reality on the statistical level (to which experimental verification is restricted) would be hiding the effects of a violation of (3) brought about on the individual level. Such a kind of nonlocality would have properties reminiscent of the strange properties of the world aether, and is bound to be rejected for analogous reasons.

Since the locality requirement (3) must only be satisfied if the measurements on particles 1 and 2 are performed in causally disjoint regions of space–time, a real test of this condition must involve a measurement setup in which any information on the choice of either A_2 or B_2 is outside the backward lightcone of the A_1 measurement. For this reason Aspect [8] devised his "switching" experiment, intending to choose randomly between A_2 and B_2 at such a time that no information about this choice can reach at a subluminal velocity the (also randomly chosen) measurement arrangement for particle 1. The violation of the BI found experimentally in the "switching" experiment is widely interpreted as evidence against the locality property (3), and as a proof of the nonlocality of the quantum world.

This conclusion has been criticized on the grounds that randomness may not have been sufficiently warranted [9], or that the efficiency of the detection process is not duly taken into account [10]. These criticisms may be justified but will not be discussed here because they refer to the practical performance of the experiment. We have no reason to expect that the experiments, if performed properly, will yield results that are different from the quantum mechanical ones. Hence, we will start from the assumption that certain EPR experiments will be able to violate the BI.

3. ADDITIONAL ASSUMPTIONS

Let us indicate the assumption of locality by LOC. By ∼LOC and ∼BI we indicate the possibility that LOC or BI may be violated. The implication

$$\sim \text{BI} \rightarrow \sim \text{LOC} \tag{5}$$

is equivalent with

$$\text{LOC} \rightarrow \text{BI}.$$

Of course, violation of locality can only be inferred from an experimental violation of the Bell inequalities if in the derivation of BI the assumption LOC is the *only* assumption. If an additional asssumption (ADD) would have been used in the derivation of BI, we would have

$$\text{LOC} \cap \text{ADD} \rightarrow \text{BI},$$

or

$$\sim \text{BI} \rightarrow (\sim \text{LOC}) \cup (\sim \text{ADD}),$$

implying that the violation of BI might be blamed either on a violation of locality or on a violation of the additional assumption. Only if a derivation would not involve any additional assumption, the reasoning (5) would be a cogent one. In this section we discuss a number of additional assumptions that are used in derivations of the Bell inequalities.

3.1 COUNTERFACTUAL DEFINITENESS

An additional assumption sometimes [11] employed in deriving BI is counterfactual definiteness. By CFD we mean that a quantum mechanical observable has an objective and well–defined value not only if the observable is measured but also if no measurement is made, or even if another, possibly incompatible, observable is measured instead. Thus, if in the EPR experiments of fig. 1 the pair (A_1, A_2) is actually measured, and the measurement result (a_{1i}, a_{2k}) is obtained, then CFD implies that we can also attribute to B_1 and B_2 well–defined values b_{1j} and $b_{2\ell}$, respectively. These are the values we would have obtained if we had measured the pair (B_1, B_2) instead of (A_1, A_2). From the existence of the quartets $(a_{1i}, b_{1j}, a_{2k}, b_{2\ell})$ the BI can easily be derived.

As discussed in de Muynck and De Baere [13] locality cannot be the only reason for the existence of quartets of measurement results. CFD is an *additional* assumption over the locality assumption. It seems to us that Stapp's disagreement [12] with our conclusion follows from his idea that CFD cannot be satisfied within quantum mechanics *because of nonlocality.* Thus, a violation of the locality condition (3),

$$a_{1i}(A_1, A_2) \neq a_{1i}(A_1, B_2) \tag{6}$$

is at the same time a violation of CFD, because then, evidently, the value of A_1 is different in an (A_1, A_2) measurement from its value in an (A_1, B_2) measurement.

This reasoning would be a cogent one if violation of LOC would be the *only* possible explanation of the violation of CFD. However, there exists another possible explanation, based on the notion of complementarity as embodied in Heisenberg's disturbance theory of measurement. According to this theory a measurement of observable B_1 disturbs observables A_1, incompatible with B_1. This implies that in the context of a B_1 measurement the observable A_1 has a different value from the one we would have obtained if A_1 itself was measured. Thus

$$a_{1i}(A_1) \neq a_{1i}(B_1). \tag{7}$$

Also from this inequality it follows that it does not make sense to attribute a uniquely defined quartet of measurement results to an individual preparation of the particle pair, thus blocking a derivation of the BI (see [3] for a comprehensive account).

Of course, it is not possible to prove rigorously that it is (7) rather than (6) that is preventing CFD. From the viewpoint of plausibility, however, the choice is not difficult. Whereas (6) refers to some nonlocal interaction the existence of which does not have any independent support, the explanation through (7) is based on the same ideas about the fundamental role played by the (local) interaction between object and measuring instrument which led Bohr to his notion of complementarity and Heisenberg to his uncertainty relations. It seems to us that the additional assumption inherent in CFD is essentially the assumption that quantum mechanical observables can be attributed to the object as *objective* properties, *not influenced in any way by the process of measurement.* The possibility of deriving BI can be attributed to this feature rather than to the locality assumption, the latter being just a special case of CFD since independence of *any* measurement interaction is a stronger assumption than the no–nonlocal–influence assumption of LOC.

3.2 COMPLETENESS OF QUANTUM MECHANICS

As another additional assumption employed over the assumption LOC in a derivation of BI we want to mention the assumption of the completeness of the quantum mechanical description. One instance in which this additional assumption is made is Bell's careful derivation based on his theory of local beables [14]. In this derivation Bell demonstrates to be aware of the possibility that quantum mechanics may not admit a simple realist interpretation in the sense that the wave function be a description of something "really existing". According to him the wave function is not a 'beable', beables supposedly describing entities that do really exist. As examples of beables Bell mentions "the setting of switches and knobs on experimental equipment, the currents in coils, and the readings of instruments". According to him "'observables' must be made, somehow, out of beables".

In Bell's theory of local beables the important quantities for (quantum) physics are the conditional probabilities $p_A(a_i|\Lambda)$ of a measurement result a_i of observable A, given a specification Λ

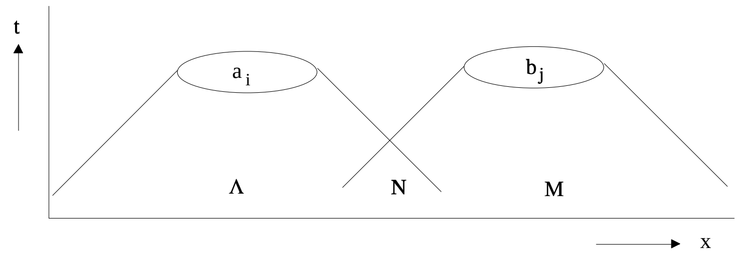

Figure 2: Backward lightcones in an EPR experiment

of "some" beables in the backward lightcone of the measurement event a_i. If Λ is not a complete specification of all beables in the backward ligthcone of a_i, then it is possible that the specification can be expanded, for instance, by a measurement result b_j of an observable B measured simultaneously with A in a different region (cf. figure 2), as is done in an EPR experiment. Since A and B may be correlated, it is very well possible that

$$p_A(a_i|\Lambda, b_1) \neq p_A(a_i|\Lambda, b_2). \tag{8}$$

Only if Λ is a *complete* specification of all beables in the backward lightcone of A we must have

$$p_A(a_i|\Lambda, B) = p_A(a_i|\Lambda), \tag{9}$$

in which B may be either a measurement result of the B–measurement or a knob setting of the instrument. More specifically, if Λ is a specification as above (i.e., not supposed to be a complete specification in the backward lightcone of a_i), and N is a *complete specification* of all the beables belonging to the overlap of the backward lightcones of the $A-$ and $B-$ measurements, then

$$p_A(a_i|\Lambda, N, B) = p_A(a_i|\Lambda, N) \tag{10}$$

(B either a knob setting or a measurement result). In this reasoning the assumption of local causality is essential in order to confine influences of local beables to their forward lightcones.

As an example Bell considers α decay of a radioactive nucleus. Let the decay process be observed by two α detectors, D_1 and D_2. Let a_i be the event "detector D_1 registers the α particle", and B the event "detector D_2 registers the α particle". If N is the complete specification defined before, then local causality implies (10). However, it is clear that (10) cannot be satisfied, since the probability that detector D_2 registers the α particle will be changing (it drops to zero) as soon as the α particle is registered by detector D_1.

It is also Bell's conclusion that from this example it only would follow that quantum mechanics is not locally causal if N is complete. Hence, either "locality" or "completeness" must be rejected. Bell's choice is here to reject completeness of quantum mechanics: the precise way the nucleus is decaying may be governed by additional *hidden* beables!

In his treatment of local hidden beables Bell considers EPR experiments as in figure 1, for instance the simultaneous measurement of the observables A_1 and A_2. Consider the conditional probability distribution $p_{A_1A_2}(a_{1i}, a_{2k}|\Lambda, M, N)$, conditional on the (hidden) beables specifications Λ, M and N in the regions indicated in figure 2. Then

$$p_{A_1A_2}(a_{1i}, a_{2k}|\Lambda, M, N) = p_{A_1A_2}(a_{1i}|\Lambda, M, N, a_{2k})p_{A_1A_2}(a_{2k}|\Lambda, M, N).$$

If N is a complete specification of the (hidden) beables in the overlap of the backward lightcones of the two measured events, then local causality would imply (10):

$$p_{A_1A_2}(a_{1i}|\Lambda, M, N, a_{2k}) = p_{A_1}(a_{1i}|\Lambda, N),$$
$$p_{A_1A_2}(a_{2k}|\Lambda, M, N) = p_{A_2}(a_{2k}|M, N).$$

Hence,

$$p_{A_1A_2}(a_{1i}, a_{2k}|\Lambda, M, N) = p_{A_1}(a_{1i}|\Lambda, N)p_{A_2}(a_{2k}|M, N). \tag{11}$$

The equality (11) is essentially the locality condition introduced by Clauser and Horne [5], from which BI are straightforwardly derived.

From this derivation often far–reaching conclusions have been drawn. Thus Clauser and Shimony [15] find the conclusion "philosophically startling: either one must totally abandon the realistic philosophy of most working scientists (i.e., reject hidden variables, dM and DB), or dramatically revise our concept of space–time (i.e., reject local causality, dM and DB)". Indeed, the mere existence of hidden beables is an additional assumption over LOC, the rejection of which offering an alternative to the rejection of LOC in order to escape the BI. For the diehard positivist this may be a solution to save local causality. Since, however, we agree with Clauser and Shimony that most working scientists do believe in the reality of the microscopic world of electrons and other elementary particles, we think that this solution will not be chosen by many. This may be the reason for the widespread idea that it is LOC that is inconsistent with quantum mechanical results violating BI. It should once more be noted, however, that this conclusion only holds if the derivation of BI along the lines of the theory of local beables is completely general, and does not rest on some hidden additional assumption not satisfied by every local hidden variables theory. It is precisely the purpose of the next section to demonstrate that there is such an additional assumption.

3.3 EXISTENCE OF CONDITIONAL PROBABILITIES WITHIN THE DOMAIN OF APPLICATION OF QUANTUM MECHANICS

In the derivation of BI using the theory of local beables the existence was assumed of probabilities $p_A(a_i|\Lambda, N)$ of values a_i of the quantum mechanical observable A, conditioned on a *complete* set of beables N. Often, as in the Clauser–Horne theory, the complete specification N is symbolized by one instantaneous value λ of the hidden variable (λ may stand for instantaneous values of a set of hidden variables, or even of a field quantity like the electromagnetic field). Then the question may be asked whether the existence of the conditional probability $p_A(a_i|\lambda)$ is self–evident and is true for any HV theory. We shall argue in this section that this is not the case if A is a quantummechanical observable. It is our conjecture that such conditional probabilities do not have a meaning within the domain of application of quantum mechanics, and that the additional assumption of the existence of $p_A(a_i|\lambda)$ may be playing a major role in the derivation of the BI.

The reason for this conjecture stems from the possibility of entertaining an empiricist interpretation of quantum mechanics. In such an empiricist interpretation a quantum mechanical observable is not an instantaneous property of the microscopic object, but corresponds to a pointer position of a macroscopic measuring instrument. Hence, the question is: does it make sense to consider the HV state λ at some initial time as the (probabilistic) cause of the final pointer position? If so, which moment should we take as the initial time?

It seems to us that the answer to this latter question may be as meaningless as the answer to the question in statistical thermodynamics of which instantaneous configuration $\{(q_n, p_n)\}$ of the canonical variables in phase space is responsible for the temperature of the object. In thermodynamics we are used to the concept of temperature, not as an instantaneous property of the object but as a quantity that has only a meaning in an average sense. Temperature is a property of the macrostate (the canonical density function $Z^{-1}e^{-H/kT}$); the relation between the microstate $\{(q_n, p_n)\}$ and the macrostate is, according to the ergodic hypothesis, one of time averaging. This is not to say that instantaneous properties of the object do not make sense in classical statistical mechanics. The point is, that the variables of statistical thermodynamics (like temperature, pressure, entropy, etc.) are all measured by means of physical operations taking certain time averaging. If we want to learn about instantaneous properties of the system, we will have to perform measurements that are sensitive to the instantaneous values of the microscopic variables. Such measurements may in principle not be impossible; they are, however, outside the domain of application of thermodynamics.

The assumption of the applicability of conditional probabilities $p(a_i|\lambda)$ within quantum mechanics, tacitly made in derivations of the Bell inequalities like the one by Clauser and Horne [5], is a genuine *additional* assumption. The assumption may be true for measurements that are so fast that the instantaneous value of λ is probed. In this case the Bell inequalities should be satisfied. However, it may be as impossible to perform such measurements within the domain of quantum mechanics as it is impossible to measure the instantaneous configuration of the canonical phase space variables of a thermodynamic object.

4. THERMODYNAMIC ANALOGY OF QUANTUM MECHANICS

The idea of an analogy between quantum mechanics and thermodynamics is not new, and already discussed by Bohr [16] as an example of the complementarity of thermodynamical and mechanical concepts, preventing a precise definition of the system's position in phase space if temperature is well–defined. Bohr, however, did not attempt to develop this analogy further [17] because he considered complementarity in quantum mechanics and thermodynamics as two quite different physical problems (also [18]).

A more fundamental relation between quantum mechanics and thermodynamics was assumed by de Broglie in his later work [19], in which he developed the idea of the quantum mechanical particle immersed in a "hidden thermostat", the notion of quantum mechanics as describing the state of a particle in equilibrium with some fluctuating medium being considered before by Bohm [20] and Bohm and Vigier [21]. Afterwards such ideas have been developed further by Nelson [22, 23] and many others (cf. [24]). It is not our purpose to give in this paper a review of all developments in this direction. Instead we want to stress a feature of the thermodynamic analogy that to the best of our knowledge is not observed before, viz., the possibility that quantum mechanical measurements do not probe the instantaneous hidden variable state λ, but only yield information on equilibrium states, comparable with the canonical density function of statistical thermodynamics. Stated differently, quantum mechanical measurements might be measurements of quantities that are comparable with thermodynamic quantities, in the sense that an *individual* measurement result a_i can not be considered as referring to a specific value of λ, but is the time–averaged result of an observation process that takes a certain amount of time for the measuring instrument to obtain sufficient information on the object in order to direct a pointer to its final position. If this is correct, then conditional probabilities $p(a_i|\lambda)$ do not have a meaning within the domain of quantum mechanics, and proofs based on such conditional probabilities are inapplicable within this domain.

It was claimed by Eberhard [38] that the expression

$$p(a_i) = \int d\lambda \; \rho(\lambda) \; p(a_i|\lambda) \tag{12}$$

is the most general one to be obtained in a HV theory. If this would be true, then the locality condition (11), generally written as

$$p(a_i, b_k|\lambda) = p(a_i|\lambda)p(b_k|\lambda), \tag{13}$$

inevitably causes the BI to be satisfied. Note that the conditional probabilities may be dependent on the measurement procedure [39], so invoking contextualism [40] is not sufficient to evade the BI: the probability distribution $\rho(\lambda)$ is (in the local theory) the distribution as prepared in a space–time region that is *not* in the forward lightcones of the measurement events, and, hence, is independent of the measurements; all influence of the measurement is thought to be inherent in the conditional probabilities $p(a_i|\lambda)$ and $p(b_k|\lambda)$. Hence, if Eberhard's claim would be right, contextualism would not be very helpful as an additional assumption over the locality assumption.

Yet, we think that contextualism may be an important element if quantum mechanics, analogously to thermodynamics, is a theory of equilibrium processes. As a matter of fact, contextualism is important also in thermodynamics, since the canonical density function $Z^{-1}e^{-H/kT}$ is dependent on the experimental arrangement through the potential energy of the system, which should be taken ∞ outside the boundaries of the container. It is important to note that the thermodynamic states are states *in equilibrium both with the heat bath specifying temperature* T *and with the container specifying the region where* $Z^{-1}e^{-H/kT} \neq 0$. Hence, whereas the microstate $\{(q_n, p_n)\}$ may be considered as an objective (i.e., noncontextual) description of the object, is the thermodynamic macrostate $Z^{-1}e^{-H/kT}$ not only of a statistical nature, to be connected with the microstates through the ergodicity property, but this connection moreover is dependent on the experimental arrangement: in principle thermodynamic properties are dependent on the experimental context.

If quantum mechanics can be considered as a theory of equilibrium processes analogous to thermodynamics, then we should also distinguish between equilibrium and non–equilibrium states of the HVs. The HV state λ could be comparable to the dynamic microstate $\{(q_n, p_n)\}$ of statistical thermodynamics, realizing Einstein's ideal of an objective description of the object, independent of any measurement to be performed. However, like the dynamic microstates of statistical thermodynamics, these HV states λ need not play a decisive role in a quantum mechanical measurement. Such measurements may be sensitive only to certain *statistical* states of the HV, which we shall indicate as $\overline{\lambda}$ in order to exhibit that these states are connected with the dynamic states by means of (ergodic) time averaging. These latter states are comparable with the microstates of thermodynamics, a state $\overline{\lambda}$ being observationally equivalent with a statistical "macro"state describing an ensemble of "micro"states analogous to the canonical ensemble of thermodynamics. If this analogy is valid then quantum mechanics would only probe such states in which the HVs are in equilibrium with the measurement arrangement. Bohr's claim of the completeness of quantum mechanics could be understood in the sense that the measurements we are able to perform can not yield any information transcending the one contained in these "macro"states. Stated differently: it does not make sense to conditionalize measurement results of quantum mechanical measurements on HV "micro"states λ; only a conditioning on the "macro"states $\overline{\lambda}$ has an observational meaning within the domain of quantum mechanics. Hence, the conditional probabilities $p(a_i, b_j|\lambda)$ used in the derivations of the BI are not applicable within the domain of quantum mechanics; we should use $p(a_i, b_j|\overline{\lambda})$ instead.

The distinction between HV "micro" and "macro"states would not be of much use in preventing a derivation of the Bell inequalities for EPR experiments if it would be possible to replace the HV "micro"state λ in the derivation by the HV "macro"state $\overline{\lambda}$. Such a replacement seems certainly justified in the locality assumptions (11) and (13), since only in this way does it seem possible to reproduce the statistical locality of quantum mechanical measurements performed in causally disjoint regions. Thus, we must have

$$p(a_i, b_j|\overline{\lambda}) = p(a_i|\overline{\lambda})p(b_j|\overline{\lambda}). \tag{14}$$

On the basis of this assumption BI could be derived completely analogously to the Clauser–Horne derivation if the "macro"state $\overline{\lambda}$ could be taken the same in all four EPR measurements. This, however, cannot be the case if our thermodynamic analogy is valid. The reason for this is the contextualism of the HV "macro"states $\overline{\lambda}$. It seems to us that the essential role of the measurement arrangement, always stressed by Bohr, should be understood primarily in the sense that, even if it would be possible to prepare the object in the *same* HV "micro"state λ before measuring either one of two incompatible observables A or B, then the "macro"states $\overline{\lambda}$ that can be used in the conditional probabilities $p(a_i|\overline{\lambda})$ and $p(b_j|\overline{\lambda})$ should be *different* because there is equilibrium with different incompatible measurement arrangements. In order to make this explicit we write $p(a_i|\overline{\lambda}^A)$ and $p(b_j|\overline{\lambda}^B)$.

In derivations of BI along the Clauser–Horne line it is essential that the same initial HV state can be taken in all four EPR experiments involved. This was called a reproducibility hypothesis (RH) in earlier work [4, 25, 26]. It was stressed that RH is a hypothesis that is possibly not justified within the domain of quantum mechanics. In the present paper we conjecture a possible physical explanation of this failure of RH, based on one of the most fundamental ideas that led Bohr to his notion of complementarity, viz., the idea of the essential influence of the measurement arrangement in understanding quantum mechanical phenomena. Bohr, however, did not draw a sharp distinction between the preparation and the measurement parts of a quantum mechanical measurement arrangement. By exploiting the thermodynamic analogy we may be able to distinguish better between these elements, and value the possible role played by the *measurement* arrangement in the *preparation*, to the effect that

$$\overline{\lambda}^A \neq \overline{\lambda}^B \tag{15}$$

if A and B are incompatible. If this is correct, then it is impossible to reproduce the same HV "macro"state in different (incompatible) EPR experiments. This is sufficient to block a derivation of BI. It is rewarding that this blocking is caused by the same peculiarity of quantum mechanics, viz., possible *in*compatibility of observables that is at the very core of this theory.

In thermodynamics fluctuations of a microscopic quantity, for instance energy [27], are the origin of the difference

$$\overline{\mathcal{E}^2} \neq \overline{\mathcal{E}}^2,$$

even though the thermodynamic energy has a well–defined value. If quantum mechanical measurement outcomes do not correspond with instantaneous properties of the object, but should be compared with thermodynamic quantities like temperature and pressure, which are determined by means of averaging over a certain space–time region, we may expect analogous inequalities. This is how the analogy with thermodynamic systems suggests in what sense another proof of the impossibility of hidden variables, viz., the proof by Kochen and Specker [28], is based on assumptions that may be too strong, and can be relaxed so as to prevent the proof from being applicable to quantum mechanical systems. As a matter of fact, in the association of quantum mechanical observables A and hidden variables f_A an important assumption, made by Kochen en Specker, is that the hidden variable $g(f_A)$ corresponds with the observable $g(A)$, i.e.,

$$g(f_A) = f_{g(A)}. \tag{16}$$

For eigenstates of A we have $\langle A \rangle^2 = \langle A^2 \rangle$. In a hidden variables theory satisfying the thermodynamic analogy this would imply $\overline{f_A}^2 = \overline{f_A^2}$, which, evidently, does not generally hold in thermodynamics. For this reason the Kochen–Specher requirement (16) is too restrictive. This requirement stems from the idea that the value of a quantum mechanical observable A is an objective property of the object, any function $g(A)$ of it to be found simply by means of mathematical manipulation. Thus, any function $g(A)$ of A is thought to be counterfactually determined by A. Evidently, the requirement (16) insufficiently takes into account the fact that measurement is a physical process, and that, even in an eigenstate of a quantum mechanical observable, the determination of its value may be thought of as a measurement of *an average value* of a random variable.

5. NONLOCALITY OF QUANTUM MECHANICS

Contrary to the often stated assertion that nonlocality is proven experimentally by the results of the Aspect switching experiments [8], these experiments are corroborating the locality of quantum mechanics as embodied in the principle of local commutativity, stating that observables measured jointly in causally disjoint regions of space–time should correspond with commuting observables. It seems to us that, in view of this fact, a nonlocal reality underlying the domain of quantum phenomena would be utterly improbable. As discussed in the previous sections, violation of the Bell inequalities can be attributed to additional assumptions in the derivation of the inequalities.

Although quantum mechanics satisfies the postulate of local commutativity it cannot be considered a local theory. This is caused by problems encountered in the attempts at unifying quantum mechanics and relativity theory. The principle of local causality is at the basis of the latter theory. Physical influences, according to this principle, propagate from one space–time region to another one at velocities not exceeding the velocity of light. Superluminal spreading of the solutions of the (nonrelativistic) Schrödinger equation is well known. Less well–known is that superluminal spreading continues to appear in *relativistic* quantum mechanics. As a matter of fact, according to a theorem by Borchers and Schlieder [29] it is a property of any quantum mechanical propagation generated by a Hamiltonian having a spectrum that is bounded from below (also [30]). A system of this kind, even if it is relativistic, could be used to propagate effects at a superluminal speed. And, although the effects are of a statistical nature, if they would be real, they might be employed to transfer information superluminally if relative frequency is measured in a sufficient number of parallel experiments. Additional arguments for the nonlocality of quantum mechanics were given in [31].

Although, as demonstrated by Hegerfeldt and Ruijsenaars [32], the violation of causal propagation is an extremely small effect, and might for this reason be considered as empirically irrelevant [33], we want to take it seriously because it seems to us that it lends considerable support to the thermodynamical analogy discussed in sect. 4. As a matter of fact, there is a great similarity between the behaviour of solutions of the Schrödinger equation in quantum mechanics and solutions of the Boltzmann equation or the diffusion equation, the latter describing the space–time behaviour of thermodynamic quantities. Superluminal propagation of the solutions of these latter equations has never been considered a serious problem because they are interpreted as yielding *macroscopic* (phenomenological) descriptions of physical transport phenomena (e.g. [34]). These equations only have an *approximate* validity, their applicability being limited by the requirement of the existence of molecular chaos determining the Markovian character of the underlying stochastic process ([34], chapt. 4.5).

If the thermodynamic analogy discussed in sect. 4 has any relevance to quantum reality, then quantum mechanical processes might be comparable to thermodynamic processes in which the condition of molecular chaos is sufficient for the maintenance of a local equilibrium in the system. Quantum mechanical processes would then be processes in which states of (dynamic) local equilibrium evolve in such a (quasi–static) way that they always remain states of local equilibrium. Stationary solutions of the Schrödinger equation would be comparable to thermodynamic states of *global* equilibrium. The nonlocal effect, observed by Greenberger [35] if a box holding a particle at its center is expanded adiabatically by moving one of the walls (the expansion causing a displacement of the particle's wave function though the particle has not been near the wall), could equally well be understood in a local way using the thermodynamic analogy. Thus, the adiabatic adaptation of the wave functions to the changing boundary conditions could be understood as a consequence of *local* processes adapting local equilibrium to the global changes in the environment.

Hence, quantum mechanics like thermodynamics, would only be applicable to physical processes under the condition that a state of local equilibrium can be maintained during the whole process. Such states would correspond with the "macro"states $\overline{\lambda}$ introduced in sect. 4. The time averaging involved in the definition of these "macro"states would then be responsible for the "noncausal" behaviour of these states.

Nonlocality of a certain theory does not imply necessarily the nonlocality of the reality described by it. Thus, the theory of rigid bodies is a nonlocal theory since it assumes propagation at infinite velocity within the bodies. The nonlocal connection of different parts of the same rigid body does not imply a violation of causality, but is just a restriction on the states that are allowed within the domain of applicability of the theory. Analogous remarks hold with respect to thermodynamics, which is a nonlocal theory as well because thermodynamic states refer to space–time regions not smaller than the regions that are averaged over in order to define the "macro"states $Z^{-1}e^{-H/kT}$. Of course, it is not impossible to change such a state locally, for instance, by increasing the energy of one single particle, thus causing the phase–space distribution to deviate from the Maxwell–Boltzmann distribution. This, however, would lead us outside the domain of application of thermodynamics. Within this domain only such operations are allowed that maintain the physical condition of local equilibrium. Such operations must necessarily be nonlocal. This, however, does not imply that local operations, violating the equilibrium condition, would be impossible. Nor are thermodynamic quantities the only quantities that can be measured. Deviations from thermodynamical predictions could be expected, for instance, if both preparation and measurement would be performed well within the relaxation time in which the state of the system returns to equilibrium after a disturbance.

If quantum mechanical measurements only probe hidden variables "macro"states, it would not be surprising that also quantum mechanics is a nonlocal theory. Once again, this does not imply that the underlying reality must of necessity be nonlocal. It is very well possible that the underlying reality obeys a local "microscopic" theory, the apparent nonlocality of quantum mechanics being related to particular conditions to be satisfied by experiments that can be performed within its domain of application. Thus, the alleged nonlocality of quantum mechanics that is sometimes thought to be apparent in the two–slit experiment (the particle, traversing one slit, "knows" whether the other slit is open or closed), can be understood in a local way if the microscopic physical condition obtaining in a slit would be of the equilibrium type discussed in sect. 4, the equilibrium configuration being different if the other slit is opened or closed. Deviations from quantum mechanics could be expected if a particle traverses a slit, after the other slit has been opened or closed, *within the relaxation time necessary to establish equilibrium between the two slits.*

6. CONCLUSIONS

Nonlocality of quantum mechanics can have two quite different meanings: i) nonlocality of the *reality* underlying quantum mechanics, discussed in connection with EPR experiments and often assessed as ensuing from a violation of the Bell inequalities; ii) nonlocality of the quantum mechanical *formalism*, showing itself in the superluminal propagation of solutions of quantum mechanical evolution equations and the Borchers–Schlieder theorem.

In the present paper we have discussed arguments why the first kind of nonlocality may be an issue based on an overestimation of the role played by the assumption of locality in derivations of the Bell inequalities. Additional assumptions are equally important. Several such additional assumptions have been investigated in this paper, of which, to our knowledge, the one of sect. 3.3 has not been discussed before. We have argued in favour of the assumption that the existence of conditional probabilities $p(a_i|\lambda)$ is not justifiable within the domain of application of quantum mechanics. Relaxation of this assumption suggests a model of reality, underlying quantum mechanics, that is very analogous to the reality underlying thermodynamics, and that is sufficient to block derivation of the Bell inequalities. In particular, a distinction between "micro"states λ and "macro"states $\overline{\lambda}$ offers the possibility of introducing probabilities $p(a_i|\overline{\lambda})$, conditioned on "macro"states $\overline{\lambda}$ that are in equilibrium with the measuring instrument.

The thermodynamic analogy also suggests a way to deal with the second kind of nonlocality mentioned above. It must be stressed that this kind of nonlocality is not directly related to the nonlocality issue connected with the Bell inequalities, the latter issue giving rise to an interpretation of quantum mechanics as an exact and local/causal description of a nonlocal/noncausal underlying reality. Quantum nonlocality as a consequence of the thermodynamic analogy, on the contrary, characterizes quantum mechanics as an approximate nonlocal/noncausal description of a (possibly) local/causal underlying reality. As discussed in sect. 4 the violation of the Bell inequalities by quantum mechanics may be seen as a consequence of the *local* equilibrium of the state of each of the particles in an EPR experiment with the measuring apparatus it is directly interacting with. Only if the measurement would be able to probe the "micro"state λ instead of the "macro"state $\overline{\lambda}$, it could be expected that the Bell inequalities are satisfied. Such experiments would have to be performed well within the relaxation time of "subquantum" processes leading to the "equilibrium" states $\overline{\lambda}$. By Bohm [36] this time is estimated "for the sake of illustration" to be $10^{-13}/c$ seconds, c the velocity of light. If this is correct, then such experiments are far outside the range of present experimental possibility. In an electron two–slit experiment with distance $d = 1\mu$ between the slits [37], if one slit is opened and closed alternately at a frequency $d/c \sim 10^{14}$ Hz, no global equilibrium between the two slits could establish, and deviations from quantum mechanical predictions are to be expected. On the other hand, a switching frequency of 50 MHz as in the Aspect switching experiment [8] does not give any reason to expect deviations from quantum mechanical predictions.

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NOTES

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