Interpretations of quantum mechanics, joint measurement of incompatible observables, and counterfactual definiteness

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The validity of the conclusion to the nonlocality of quantum mechanics, accepted widely today as the only reasonable solution to the EPR and Bell issues, is questioned and criticized. Arguments are presented which remove the compelling character of this conclusion and make clear that it is not the most obvious solution. Alternative solutions are developed which are free of the contradictions related with the nonlocality conclusion. Firstly, the dependence on the adopted interpretation is shown, with the conclusion that the alleged nonlocality property of the quantum formalism *may have been reached on the basis of an interpretation that is unnecessarily restrictive. Secondly, by extending the conventional quantum formalism along the lines of Ludwig and Davies it is shown that the Bell problem may be related to complementarity rather than to nonlocality. Finally, the dependence on counterfactual reasoning is critically examined. It appears that locality on the quantum level may still be retained provided one accepts a newly proposed principle of nonreproducibility at the individual quantum level as an alternative of quantum nonlocality. It is concluded that the locality principle can retain its general validity, in full conformity with all experimental data.*

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1. Introduction

From a purely pragmatic point of view quantum mechanics (QM) is a highly satisfactory theory. Its domain of application is really enormous. All attempts, from Einstein (cf. [(1)]) to Aspect [(2, 3)], to frame experiments yielding results violating QM have been unsuccesful up till now. QM always turned out to be applicable. Bohr's thesis of the completeness of QM seems to be corroborated thus far.

Yet, there exist a number of reasons why we might feel a certain uneasiness over the present status of QM. First, there is the proliferation of different interpretations of the quantum formalism, ranging from the purely empiricist interpretation looking upon the formalism as a description of the (cor)relation between macroscopic preparation acts and equally macroscopic measurement phenomena, to a many–worlds interpretation in which the universe is thought to multiply itself into an infinite set of parallel worlds. Such a proliferation of paradigms is characteristic of a Kuhnian crisis in the development of theory, asking for in–depth analysis rather than acquiescence in the status quo.

A second reason, like the former one of a methodological nature, is the observation that in the history of physics before QM no theory turned out to be universally valid. Every single theory was valid only if applied to a restricted part of reality, its domain of application. Do we have any reason to believe that QM is different, and will hold true for whatever future experiments we may be able to think of? For instance, will the Heisenberg uncertainty relations be as generally valid as was thought by Bohr and Heisenberg to be the case? Or did they restrict their considerations to those physical situations that are well within the domain of quantum mechanics, thus essentially presupposing what they wanted to prove? What can we learn from theories purporting to generalize QM?

A third reason is related to the nonlocality problem as emerging from Bell's theorem. It is often stated that nonlocality is proven experimentally by the results of the Aspect switching experiments [(3)]. On the other hand, these experiments are well described by quantum mechanics. They, hence, are corroborating the locality of quantum mechanics as embodied in the principle of local commutativity, to the effect that the probability distribution of the results of a measurement cannot be influenced by another measurement performed in a causally disjoint region of space-time. So, if the violation of the Bell inequalities (BI) would be a consequence of a nonlocal reality underlying the quantum world, then it would require from nature a rather conspiratory attitude preventing any trace of this nonlocality from being observed at the (physically relevant) level of the relative frequencies of quantum mechanical measurement results. For this reason it seems reasonable to scrutinize the presuppositions on which the conclusion of nonlocality is based. What role is played by the interpretation that is entertained? Is it possible to attribute the violation of the BI to other assumptions, not explicitly mentioned, or to other features of QM than the ones usually taken into account? In the following two such features will be discussed extensively, viz., i) the mutual disturbance of measurement results in a joint measurement of incompatible observables, and ii) a principle of nonreproducibility restricting the justified use of counterfactual reasoning in QM. It is the purpose of the present paper to show that conclusions with respect to (non)locality, drawn from the EPR problem or from a violation of the Bell inequalities, may stem either from too stringent an interpretation of the quantum formalism, or from too restrictive an application of the formalism itself.

In sect. 2. we first review briefly the discussion between Bohr and Einstein on the EPR problem, in order to conclude that the outcome of this discussion is strongly dependent on the interpretation. Two broad classes of interpretations are distinguished, viz., the empiricist and the realist ones, the latter class being subdivided into an objectivistic and a contextualistic species. The relative merits of these interpretations are discussed. In this section we also consider the interpretational problems voiced by Ballentine [(4)] with respect to the Heisenberg uncertainty relations. Do these relations describe a disturbance caused by the measurement procedure as maintained by Heisenberg, or must they be considered as fundamental limitations set by nature on the preparation of the object within the domain of quantum mechanics, independently of any measurement? It appears that no clearcut answer to this question can be given on the basis of the Dirac-von Neumann formulation of QM, precisely because this formalism is not obviously inconsistent with a realist interpretation in which measurement results are interpreted as properties of the microscopic object, possessed either objectively or contextualistically. In this view of quantum mechanics it is difficult to evaluate the role of the measuring instrument in the measurement process, this role seemingly being restricted to a possible source of disturbance.

In sect. 3. we review a generalization of the Dirac-von Neumann formalism, due to Davies [(5)] and Ludwig [(6)], in which the active role of the measuring instrument in bringing the microscopic information to the observational level is taken into account. In this generalized formalism quantum mechanical observables do not correspond with self-adjoint operators but with positive operator-valued measures, thus enabling to describe a wider class of measurements than can be covered by the Dirac-von Neumann formalism. In particular, in the generalized formalism it is possible to describe measurements that can be interpreted as joint measurements of incompatible Dirac-von Neumann observables [(7)], such measurements being impossible from the point of view of the more restricted formalism. It turns out to be possible to corroborate the idea of complementarity as developed by Bohr and Heisenberg in their discussions of the "Gedanken" experiments (i.e., complementarity in the sense of a mutual disturbance due to the mutual exclusiveness of the measurement arrangements of incompatible observables) as a fundamental property of the generalized observables. Since this can be implemented by means of an inequality different from the Heisenberg uncertainty relation, only depending on the measurement procedure and completely independent of the preparation, this result can be seen as an endorsement and a strengthening of Ballentine's abovementioned criticism. Further implications of the generalized formalism for the interpretation are discussed in sect. 5..

In sect. 4. we discuss the problem of the Bell inequalities within an empiricist setting, this being made possible within the generalized formalism by its ability to describe the joint measurement of the four (incompatible) observables that are involved. As an example we consider a variation of Aspect's switching experiment, in which the switching elements are replaced by semi-transparant mirrors. Because of the existence of a quadrivariate joint probability distribution, the measurement results of this experiment must satisfy the Bell inequalities. By means of the Wigner measure, defined for this experiment, it is possible to relate its measurement results to the results obtained in the usual EPR experiments like the Aspect ones. In this comparison (non)locality does not seem to play any role, the difference between the experiments being reducible to complementarity.

In sect. 6. the problem of the Bell inequalities is considered from the point of view of hidden variables theories. Apart from the distinction between deterministic and stochastic hidden variables theories the possibility is also taken into account that the quantum mechanical measurement result must be considered as a property of the measuring instrument rather than as a property of the microscopic object. By this latter distinction it is possible to link on either to the empiricist or to a realist interpretation of the quantum mechanical formalism. The possibility is discussed that also in the context of hidden variables theories the problem of the Bell inequalities is connected with the existence of a quadrivariate joint probability distribution. It is demonstrated that locality can play a role in the construction of such a quadrivariate jpd for the usual EPR experiments if an additional assumption is made, viz., that it is possible to prepare the object in the *same* initial HV state in the different EPR experiments that must be performed in order to test the Bell inequalities.

In sect. 7. the need and the virtue of the introduction of hidden variables in the quantum domain is discussed and the justified use and the physical relevance of counterfactual reasoning in physics in general and in QM in particular is critically investigated. One basic criterion in this respect is that hypothetical situations should not only be allowed by the theory, but also be actualizable in real experiments. It appears, then, that this justification requires a number of general conditions to be fulfilled such as the availability of a theory and the validity of a hypothesis which may be called the "reproducibility of initial conditions". When applied to classical mechanics, it is shown that these conditions are generally valid and, hence, that CFD has general validity in the classical domain. However, when considering the issue of the nonlocality of QM, it becomes clear that QM is required to be applied in a domain where the general conditions just mentioned are not a priori valid. It is argued, therefore, that it is the alleged general validity of counterfactual reasoning in the whole domain of quantum processes that is one of the main additional assumptions mentioned above. The EPR and the Bell argumentations are reconsidered from this point of view. It is concluded, finally, that locality on the individual quantum level may be restored and the conflict with the principle of local commutativity be removed by introducing a basic quantum nonreproducibility.

2. ON THE ROLE OF INTERPRETATIONS IN THE EPR PROBLEM AND IN HEISENBERG'S UNCERTAINTY PRINCIPLE

2.1. EPR and locality

The Einstein–Podolsky–Rosen (EPR) problem [8] must be seen as an ultimate attempt at falsifying Bohr's completeness thesis of QM. Earlier proposals by Einstein had always been rebutted by Bohr by taking recourse to the interaction of object and measuring instrument. The EPR proposal was meant to consider a situation in which this interaction is absent, at least if it is assumed that sufficient spatial separation between object and measuring instrument has this effect. A system of two particles was considered, particle 1 and particle 2, say, being in a simultaneous eigenstate of the compatible observables $p_1 + p_2$ and $x_1 - x_2$. It is assumed by EPR that particles 1 and 2 are far apart, and, hence, particle 2 is not influenced by any local measurement performed on particle 1: we may either choose to measure p_1 or x_1 without changing the reality of particle 2. Because of the exact correlations inherent in the state of the two–particle system, we can obtain in this way a precise value of either p_2 or x_2. If, as assumed by EPR, quantum mechanics yields a description of *objective* reality, particle 2 must have possessed these two values before, and, hence, independently of the measurements performed on particle 1. They are so–called "elements of physical reality". Since the formalism of QM does not contain any mathematical element allowing an interpretation as a *joint* element of physical reality of two incompatible quantities, their conclusion is that QM is incomplete.

In his answer to the EPR challenge Bohr [(9)] criticizes the EPR notion of "element of physical reality" as an ambiguous one. According to him it is not allowed to interpret the quantum mechanical formalism without taking into account the whole experimental arrangement. Hence, an interpretation of quantum mechanics as a description of objective reality would not make sense; if a realist interpretation is possible at all it must be a contextualistic one. By Einstein [(10)] this answer is interpreted as signifying that in the system under discussion particle 2 can have the element of physical reality p_2 only in the context of a p_1 measurement, and, analogously x_2 in the context of an x_1 measurement. According to this way of understanding Bohr's answer completeness would be restored because of the impossibility of simultaneously measuring x_1 and p_1 (see also [(11)]). This would imply that particle 2 has no element of physical reality at all as long as no measurement is performed on particle 1, but it may obtain one instantaneously as soon as a measurement on the other particle is performed. Since this would imply a nonlocal interaction, it is Einstein's conclusion that completeness of QM implies nonlocality (note that, according to this reasoning, nonlocality does not follow if QM would be incomplete).

It is not at all clear whether Bohr would have to subscribe to Einstein's conclusion. This latter conclusion has given rise to Jammer's claim (Ref. 12, p.197) that the EPR problem obliged Bohr to change his contextualistic interpretation of quantum mechanics from an interactionalist to a relationalist one in which already the mere *relation* to a distant measuring instrument is sufficient to determine the element of physical reality, without an *interaction* being necessary. From the subtle way the matter is formulated by Bohr viz., " no question of a mechanical disturbance", but "an influence on the very conditions which define the possible types of *predictions* regarding the future behavior of the system" [(9)] (emphasis ours) it might be surmised that according to Bohr an element of physical reality of particle 2 makes sense only in the context of a measurement of this *very* particle. If this reading of Bohr would be correct, it is the impossibility of measuring simultaneously x_2 and p_2 (rather than x_1 and p_1) that prevents the values of x_2 and p_2 from being simultaneously determined. In that case the local interaction between particle 2 and the measuring instrument for either x_2 or p_2 can, once again, be taken recourse to by Bohr. No change of interpretation would be necessary in that case. Neither would nonlocality follow from completeness.

An understanding of Bohr's views along these lines would bring him in line with those adherents of empiricism who propose to see QM as a description of measurement phenomena (observables) only (e.g. Wheeler [13]), without any relevance to a reality that is not observed by means of a well–defined measurement arrangement. Indeed, if QM is about such observables only, the whole idea of the EPR element of physical reality evaporates as a quantum mechanical concept. It was also noticed [14, 15] that in this view locality is warranted by the postulate of local commutativity, expressing the compatibility of observables measured in causally disjoint regions of space–time.

As discussed by de Muynck [16] it is far from clear from Bohr's writings whether an empiricist view can be attributed to him in the above-mentioned sense. Bohr considered the quantum mechanical wave function as a mere symbolic tool for calculating probabilities, and, hence, his interpretation of the wave function is certainly not a realist one. However, as regards observables he seems to think in a much more realist way [17]. Due to his insistence on a classical description of measurement, physical quantities like position and momentum obtain a very classical meaning, to be understood in classical terms. Although with Bohr position is only defined in the context of a position measurement, and, hence, to a particle a value of this quantity can be attributed only under the latter circumstances, it is important to note that this value nevertheless is seen by him as a *property of the object.* Hence, as far as measurement results are concerned Bohr does not seem to entertain an empiricist interpretation. On the contrary, in this respect his interpretation is a more realist one, be it of a contextualistic nature. This realism may have seduced Bohr into following EPR quite a bit further than would be allowed on the basis of a pure empiricism. He did not deny the element of physical reality x_2 any right of existence as long as observable x_2 is not measured, but he merely pointed at an ambiguity in its definition that could be lifted by referring to the distant measurement setup of particle 1. We shall return to this in sect. 5.. Here we want to stress that evidently the question of the nonlocality of quantum mechanics hinges strongly on the interpretation that is adopted.

2.2. Realist and empiricist interpretations of QM

With respect to the interpretation of the mathematical formalism of quantum mechanics two broad classes can be distinguished, viz., the empiricist[3] and the realist ones. In the empiricist interpretation the formalism does not describe reality as such. It only serves to calculate probabilities (relative frequencies) of certain phenomena that can be interpreted as corresponding with the results of a quantum mechanical measurement. The probabilities are conditioned on certain procedures, to be interpreted as quantum mechanical preparation procedures. Thus, the wave function or density operator can

[3]Often an "instrumentalist" interpretation is juxtaposed to the "realist" one, in the former the mathematical formalism being considered as just an instrument to calculate measurement outcomes. Our empiricist interpretation is instrumentalist in this sense. However, by referring to empiricism we want to stress the direct observability of the measurement results of quantum mechanics, this latter feature not being generally inherent in the definition of instrumentalism.

be interpreted as symbolizing a preparation procedure; in the same vein an hermitian operator describes symbolically a quantum mechanical measurement. Wave function and hermitian operator are not thought to correspond to something existing in microscopic reality. They are just labels of (macroscopic) instruments that can be found in the laboratory. QM is thought to describe only (cor)relations of preparation acts and measurement phenomena. In the Dirac–von Neumann axiomatization of QM such a relation is represented by the expectation value of the hermitian operator in the state described by the density operator ρ. More specifically, probabilities p_m are given by the expectation values of projection operators,

$$p_m = Tr\rho E_m, E_m^2 = E_m, \tag{1}$$

each projection operator E_m corresponding with a pointer position of the measuring instrument. Here the set of projection operators $\{E_m\}$ is the spectral representation of the hermitian operator labeling the measuring instrument. It must be noted that in an empiricist interpretation of QM the eigenvalues of the hermitian operator do not play a significant role, because these eigenvalues do not correspond with properties of the microscopic object, but are just numbers labeling the measurement scale of the measuring instrument, which can be chosen in a rather arbitrary way. For this reason we might as well label the measuring instrument by the spectral representation $\{E_m\}$ rather than by the hermitian operator.

The empiricist interpretation has achieved great popularity because of its antimetaphysical flavor: it appeals to those who think that physics must be about observables only, and about nothing else. Hence, in this interpretation neither the wave function nor the observable must be taken as a property of the microscopic object system as is done in a *realist* interpretation of QM. In this latter interpretation the wave function is often considered as a description of *the result of* a preparation rather than as a symbolic representation of the preparation itself. Similarly, in a realist interpretation the measurement result is attributed to the object as a property of the latter. Following Bohr the quantum mechanical description of reality may be thought to hold true only within the context of a well–defined measurement arrangement, hence describe *observed* reality (i.e., interacting with a measurement device) rather than *objective* reality as Einstein would have liked to have it; yet, in a realist interpretation it is thought to be a description of microscopic reality all the same.

It is not unimportant to notice that in most literature on QM the terminology used is rather a realist one. Thus electrons "*are* in a certain quantum state", "*have* a certain momentum", are found on measurement "in a certain state", and so on. As a matter of fact, a pure empiricism as defined above seems to be not generally entertained. There may exist different reasons for this. One possibility is, that a custom we are used to in the domain of classical physics is continued in quantum mechanics, pure states being treated as the analogues of the phase space points of classical physics and density operators corresponding with probability distributions on phase space [(18)], the only difference with classical mechanics being thought to be the fact that in QM there exists a fundamental limit to dispersion, expressed by the Heisenberg uncertainty relations. A more profound possibility may be the observation that QM seems to have a wider domain of application than just preparation and measurement phenomena in the laboratory. Thus, the Pauli principle seems to be essential to an explanation of pressure in neutron stars, and of electric current conduction in metals. These can hardly be counted as "observed phenomena" in the sense of Wheeler [13], since these processes seem to take place independently of any observation. However, still more important may be the feeling that QM must tell us something about microscopic reality, i.e., about electrons and photons rather than about preparing and measuring instruments. Although from a strictly positivistic point of view electrons belong to the reign of metaphysics, not many physicists will not believe in their existence.

It is well known that a realist interpretation of the wave function may summon paradox, Schrödinger's cat and EPR being the most conspicuous ones. These paradoxes are essentially connected with the problems caused by the attempt at a realist understanding of the superposition principle. As a matter of fact, the so–called "measurement problem", having its origin in the "paradoxical" notion of a superposition of macroscopic pointer states of a measuring instrument, has this very origin. From the point of view of Bohr's empiricist conception of the wave function these problems would seem to be completely irrelevant: the empiricist needs not be bothered by the superposition of a living and a dead cat as long as we are unable to measure any observable that is sensitive to the interference terms. Although an empiricism of this kind was generally accepted in the dominant "Copenhagen" interpretation (Ref. 12, p.248), already von Neumann's treatment of the measurement problem [18] demonstrates a tendency towards a more realist conception of the wave function.

Within the class of realist interpretations we can distinguish two subclasses, viz., one considering the wave function as a description of a *single* object, the other one advocating to attribute the wave function to an *ensemble* of identically prepared objects [4]. In the former class rather drastic measures have to be taken in order to cope with the paradoxes, such as introducing "many worlds" [19], or even changing the formalism of QM [20]. The "ensemble" interpretation (called "statistical" interpretation by Ballentine [4]) seems to have less problems in this respect, although it must be realized that the non–Boolean structure of the *quantum* ensemble makes it impossible to consider QM as a statistical theory analogous to classical statistical mechanics. The ensemble interpretation as a realist interpretation does not make QM more understandable. Moreover, if, as is often implied, the wave function is thought to describe *objective* reality, i.e., an ensemble of identically prepared objects not interacting with any preparing or measuring instrument, then Einstein's conclusion of nonlocality (cf. sect. 2.1.) once again follows: in an ensemble of simultaneously performed identical EPR experiments the description of the subensemble of particles 2 corresponding with the measurement result p_1 is instantaneously and "realistically" changed into the momentum eigenstate $|-p_1\rangle$. It must be noted that, contrary to sect. 2.1., in the present treatment of EPR which hinges on a realist interpretation of the *wave function* rather than of the measurement result, completeness of QM is not essential. Nonlocality follows just from the rules of QM (whether it is complete or not), provided the wave function is interpreted realistically.

As is well known nonlocality, as obtained in the objective realist "ensemble" interpretation, does not have any observational consequences [14, 15]. This makes this interpretation less attractive[4] even though it solves a number of problems. Also in the relationalist version of the contextualistic realist interpretation as attributed by Jammer to Bohr (cf. sect. 2.1.) the problem remains how, and in what sense, the far measuring arrangement can influence the reality of an object not interacting with it. We might try to solve the nonlocality problem by switching to a *strictly interactionalistic* realist interpretation, in which the wave function describes reality *only* as far as it is *directly* interacting with a measuring instrument. In the EPR problem this would mean that reality is described as far as measurements are performed on *both* particles, thus providing for a context for them both. Such a strictly interactionalistic realist interpretation has another problem, however. Since the same wave function (or density operator) can be used in different measurements, it would seem that reality as far as described by it is *independent* of the measurement arrangement rather than depending on it. As a matter of fact, reality as far as revealed by a measurement of an observable represented by $\{E_m\}$ is described just as well by the density operator

$$\rho' = \sum_m E_m \rho E_m \tag{2}$$

that yields the same probabilities (1) for $\{E_m\}$ as ρ does. It is ρ' rather than ρ that is a candidate for a strictly interactionalistic interpretation. Indeed, if the interaction between object and measuring device is treated quantum mechanically, it is possible in certain cases to demonstrate a transition from ρ to ρ' during the measurement process. If this is true, however, then *if* ρ describes reality at all, it seems to describe reality as it is *before* the interaction with the measuring instrument starts, rather than reality in interaction with a measuring instrument.

A rejection of a realist interpretation of the quantum mechanical formalism is not, as sometimes suggested, tantamount to a denial of an objective reality (e.g., "Is the moon there when nobody looks? Reality and the quantum theory" [(22)]). It is very well possible to believe in the objective existence of atoms and electrons without being committed to the thesis that this reality is described literally by the quantum mechanical formalism. This actually seems to have been Bohr's position while attributing to the wave function only a symbolic meaning.

What has to be described in the first place by the quantum mechanical formalism is the relative frequencies of certain phenomena (like clicks in particle counters, or traces in bubble chambers), that can be interpreted as indicating measurement outcomes of certain quantum mechanical measurements. Such phenomena are to be considered as properties of the measuring instrument rather than as properties of the microscopic object. This, however, does not at all imply the ontic nonexistence of the microscopic object. This object may be thought of as having initiated the measuring process, but it often gets lost in this very process. Therefore the quantum mechanical probabilities in general refer to relative frequencies in the final state of the measuring instrument rather than of the microscopic object. In an empiricist interpretation QM *is* describing (a part of) reality, however not microscopic reality itself (i.e. the *input* of the measuring

[4] For Wheeler [(21)] this is a major indication that "the wave function is not itself real, but a purely formalistic device [...] for calculating the probability of real coincidences."

instrument), but reality as transformed and amplified by the measuring process (i.e. the *output* of the measuring instrument).

Note that this implies that "unperformed experiments have no results" [23]. Hence, in the empiricist interpretation of QM the problem of counterfactuals does not arise. We think that Bohr was right in supposing that outside the context of a concrete measurement it is *not even defined* which aspect of reality is at stake: quantum mechanical properties are not objective properties of the microscopic object.

In axiomatic QM it is not customary to draw a distinction between input and output of the measuring process. Quantum mechanical measurement results are attributed to the object as properties possessed either before, during, or even after the measurement. In sect. 5. this will be discussed more extensively. Here we restrict ourselves to the remark that a choice for an empiricist interpretation of QM need not imply obedience to the maxim "Thou shalt not think" [23]. It is possible to keep thinking about reality behind the phenomena without requiring that it is QM itself that has to describe this reality. If microscopic objects exist, they must have properties of their own. If QM does not describe these properties, we shall have to device a theory different from QM in order to fill this gap.

2.3. The Heisenberg uncertainty relations

2.3.1 Ballentine's critique

The Heisenberg inequalities

$$\Delta q \Delta p \geq \hbar/2 \tag{3}$$

are often associated with the problem of the joint measurability of incompatible observables in QM.[24] The quantities Δq and Δp in (3) are considered as measurement errors induced by the attempt at obtaining simultaneous information on both position and momentum of a particle. This view has become widely accepted following the discussions by Bohr and Heisenberg of the so–called "Gedanken" experiments,[25, 1] in which it was demonstrated that there exists a certain complementarity as regards the precisions with which position and momentum can be known. This was interpreted as a mutual disturbance of the measurements of incompatible quantities if these are measured simultaneously, the disturbance being due to a mutual exclusiveness of the measurement arrangements.

The above-mentioned interpretation of the Heisenberg inequality (3) was questioned by Ballentine.[4] Let the quantities Δq and Δp have their usual meaning as standard deviations of quantum mechanical probability distributions, analogous to (1), for the spectral representations of the position and momentum observables. Thus,

$$(\Delta q)^2 = Tr\rho\left(Q^2 - \langle Q\rangle^2\right),\ (\Delta p)^2 = Tr\rho(P^2 - \langle P\rangle^2). \tag{4}$$

Then the measurements of these observables, yielding Δq and Δp, can be performed *separately*, thus avoiding any mutual disturbance because of simultaneous measurement. Notwithstanding this, Δq and Δp as defined by (4) must satisfy inequality (3) if the preparations in both measurements are identical, i.e., if they are described by the same density operator ρ.

We agree with Ballentine that, if Δq and Δp are standard deviations of probability distributions as discussed above, there need not be any relation between inequality (3) and the problem of joint measurement of position and momentum. The Heisenberg inequality seems to be a property of the preparation or of the object (depending on the interpretation of the density operator, cf. sect. 2.2.). It is true that in (4) also observables Q and P play a role, and, hence, (3) *does* refer to measurement results of quantum mechanical observables. However, these measurements may measure either Q or P separately, without introducing any disturbance, in the Bohr–Heisenberg sense, of the observable that is actually measured. Hence, no additional indeterminateness need be introduced by the measurement procedure into either Δq or Δp over the uncertainties that can be attributed to indeterminateness inherent in the preparation procedure. It is important to stress here the possibility of seeing Δq and Δp as consequences of a fluctuating *source* (i.e., preparation procedure) rather than as stemming from the measurement procedure. If this is correct, the universal validity of the Heisenberg inequalities within the domain of QM could be attributed to the peculiar fluctuation properties of quantum mechanical sources.

Ballentine's conclusion seems to be in conflict with the essential role played by the disturbing influence of the measuring instrument in the clarification of the meaning of the Heisenberg uncertainty relations through the "Gedanken" experiments. From his analysis,[4] however, it is seen that this conflict is only an apparent one, because the inequality (3) could be interpreted as a consequence of the *preparative* aspects of the measurement arrangement.

Yet, it seems to us that Ballentine's conclusion cannot be the whole answer to the problem raised by the Heisenberg uncertainty relations. The suggestion that these can (also) be seen as expressing some limitation on the joint *measurement* of position and momentum, is emanating very powerfully from certain formulations of the "Gedanken" experiments. Thus, in order to cope with Einstein's proposal that position and momentum might be measured with accuracies exceeding (3), it is demonstrated by Bohr that a change in the measurement arrangement so as to obtain knowledge on which slit the particle passed through, would wipe out the interference pattern, thus destroying the possibility of obtaining knowledge about momentum. Yet, also this application of the Heisenberg inequalities can be reduced to a problem of *preparation*. This is so because Bohr's abovementioned reasoning is essentially based on the fact that the *screen* is a quantum mechanical object, satisfying (3) in a *preparative* sense. Hence, in this "Gedanken" experiment the measurement inaccuracies satisfy (3) *because* of the impossibility of *preparing the screen* better than allowed by the Heisenberg inequalities for the screen: measurement inaccuracy is reduced to preparative inaccuracy of the measurement arrangement.

2.3.2 Preparation and measurement and their different contributions to inaccuracy

Limitations with respect to preparation of the measurement arrangement do not seem to be the only sources of measurement inaccuracy, however. A different source may stem from state evolution during the measurement. For instance, the functioning of a photodetector having quantum efficiency ξ smaller than one ($\xi < 1$) does not seem to be limited by an essential quantum mechanical limitation to the preparation of the detector. Its inaccuracy as a measuring instrument of photon number seems to be determined to a considerable extent by the *interaction* between detector and field. For this reason it seems to us that a reduction of measurement inaccuracy to preparation as in the "Gedanken" experiment is possible only in special cases. In a general treatment of measurement inaccuracy the notion of state preparation needs not play a role at all. In sect. 3. we will review a theory of measurement inaccuracy satisfying this latter characteristic. For a more complete treatment we must refer to Ref. 7.

Here we want to stress two important points: 1) the theory can be interpreted in a purely empiricist way as discussed in sect. 2.2. We will only refer to the probabilities of certain measurement results obtained in certain measurements that are labelled by quantum mechanical observables, and obtained in the context of certain quantum mechanical state preparations that are described by quantum mechanical density operators; 2) it will be seen that both preparation and measurement may contribute to uncertainty; the first contribution is adequately described by the Heisenberg inequalities (3) or its generalizations to different observables. It will, however, be demonstrated that the second contribution allows a completely different description, viz., an inaccuracy that is a property of the observable only and is completely independent of the density operator of the object. It will be demonstrated that also these quantities satisfy an inequality, and that this inequality can be interpreted as expressing mutual disturbance in the simultaneous measurement of incompatible observables. This seems to supplement Ballentine's analysis in the sense that QM *does* have to say something about simultaneous measurement of incompatible observables, but that we have to look for a relation that is different from the Heisenberg inequality (3) in order to be able to find an appropriate representation of this notion of incompatibility. It seems to us that in the "Gedanken" experiments, possibly due to a too informal treatment, the two different contributions to inaccuracy are insufficiently distinguished, thus causing considerable confusion.

It must be emphasized that an empiricist interpretation of quantum mechanics prevents us from counterfactually attributing values to observables that are not measured. For this reason our agreement with Ballentine as regards the source of the Heisenberg inequalities does not imply a general agreement with his "statistical" interpretation in which a value of both position and momentum can always be attributed to a particle. It seems to us that the impossibility of mapping the non–Boolean structure of quantum mechanical propositions in a nontrivial way into a Boolean lattice is responsible for a break–down of this idea. In an empiricist interpretation of the formalism it is clear that the simultaneous consideration of position and momentum makes only sense in the context of a *joint* measurement of these observables, the measuring instrument possessing two pointers, one for each observable, and each measurement event yielding a reading of *both* pointers.

From the empiricist point of view during a long time it has been possible to ignore completely the problem of the joint measurement of incompatible observables because it was thought that such procedures were impossible. It is by now well known that this is not correct. Many measurement procedures are known that can be interpreted as joint measurements of incompatible observables.[26] An example will be discussed in the next section. The essential property of a joint measurement procedure of several observables is its capability of generating a *joint* probability distribution. The idea of the impossibility of jointly measuring incompatible observables was largely based on the difficulty of deriving from the quantum mechanical formalism expressions that can serve as such joint probability distributions. As is well known, a generalization of (1) to the joint measurement of observables $\{E_m\}$ and $\{F_n\}$,

$$p_{mn} = Tr\rho E_m F_n, \tag{5}$$

in which $\{F_n\}$ is the spectral representation of a second hermitian operator, makes sense only if $[E_m, F_n]_- = 0$, that is, the observables must be compatible. Within the Dirac–von Neumann axiomatization of QM there does not exist an obvious way to generalize (5) to incompatible observables. Certain rigorous mathematical results, like Wigner's theorem,[27] even suggest the impossibility of such a generalization.

In the next section we will deal with an extension of the mathematical formalism of QM allowing for mathematical expressions that can be interpreted as joint probability distributions of incompatible observables. These quantities, hence, can describe the results of joint measurements of incompatible observables. It also turns out[28] that Bohr's reasoning as regards the complete wiping out of interference if path is measured in a two–slit experiment is far too schematic. As a matter of fact, Bohr considered only the two extreme cases in which either interference or path is measured accurately, by either immobilizing the screen completely or by allowing it to move freely. It is also possible, however, to consider intermediate situations in which the screen is supported by a spring having a *finite* spring constant. Such intermediate situations correspond with joint measurements in which *both* observables are measured with finite accuracies.

It is important to notice that in an empiricist interpretation of the quantum mechanical formalism the object remains completely out of sight. For this reason disturbance in a joint measurement of incompatible observables is *not a disturbance of the object.* It is a disturbance of one pointer by the change in its functioning due to the presence of the other pointer, and vice versa. Of course, this is not to say that the object is not disturbed by the measurement. In the empiricist interpretation this latter disturbance is not thought to be described by the formalism, however. We shall return to this point in sect. 5.

3. NONIDEAL MEASUREMENTS AND JOINT MEASUREMENT OF INCOMPATIBLE OBSERVABLES

3.1. Positive operator–valued measures

In the standard Dirac–von Neumann formalism of QM a quantum mechanical observable is represented by an hermitian operator or its spectral representation (cf. (1)). In the following we will have to deal with a generalization of the concept of a quantum mechanical observable due to Davies[(5)] and Ludwig[(6)] (also Holevo[(29)]). In this generalized theory an observable corresponds[5] with a *positive operator–valued measure* (POVM), that is, a set of operators $\{M_m\}$ satisfying

$$M_m \geq 0, \sum_m M_m = I \tag{6}$$

(for simplicity we stick to discrete spectra, although the theory is easily generalized to observables having a continuous spectrum). According to this theory the relative frequencies p_m of quantum mechanical measurements are described by the expectation values

$$p_m = Tr\rho M_m \tag{7}$$

of the operators of the POVM $\{M_m\}$. If the operators M_m are orthogonal projection operators (1) is recovered, yielding the special case of a *projection–valued measure* (PVM). In general, however, the operators M_m constitute a *nonorthogonal* resolution of the identity operator. Sometimes it is assumed that orthogonality is necessary in order that the different measurement results can be distinguished from each other. It has to be remembered, however, that in an empiricist interpretation the *pointer states* rather than the *object states* have to be distinguished. In a general measurement there is no one–to–one correspondence between the pointer states and the members of a set of orthogonal object states. This follows clearly from a quantum mechanical treatment of the measuring process. Let ρ_A be the initial state of the measuring instrument and U the evolution operator of the interacting system of object and measuring instrument between the initial and final times of the measurement. If the pointer states correspond with (orthogonal) projection operators Θ_m on the Hilbert space of the measuring instrument, then the probabilities of the measurement results are given by

$$p_m = Tr_{O,A} U\rho \otimes \rho_A U^\dagger \Theta_m = Tr_O \rho Tr_A \rho_A U^\dagger \Theta_m U. \tag{8}$$

From (8) it follows that

$$M_m = Tr_A \rho_A U^\dagger \Theta_m U, \tag{9}$$

[5]In the following we will often not distinguish between the observable as a label of a measuring instrument and the POVM corresponding with it in the mathematical formalism. This somewhat sloppy language increases our facility of expression without causing any conceptual problem.

which, in general, is not a projection operator.

As an illustration consider the example of the photodetector having quantum efficiency $\xi < 1$, referred to in sect. 2.3.2. Here a pointer state corresponds with the registration of a certain number, say m, of photons. Because of the nonideality of the detector this result may stem from any photon number state $|n\rangle$ with $n \geq m$. As a matter of fact, the POVM of a photodetector having quantum efficiency ξ can be calculated directly from an expression given by Loudon[30] as

$$M_m = \sum_{n=m}^{\infty} \binom{n}{m} \xi^m (1-\xi)^{n-m} N_n, \; 0 \leq m < \infty, \tag{10}$$

in which $\{N_n\}$ is the PVM of projection operators $N_n = |n\rangle \langle n|$ on the number states $|n\rangle$. The set of operators $\{M_m\}$ can easily be demonstrated to satisfy (6).

If $\xi = 1$ we obtain from (10) the equality $M_m = |m\rangle \langle m|$. So, only the *ideal* photon detector can be described by means of the Dirac–von Neumann form of QM. The common situation in experimental quantum physics, however, is the nonideal one, for which the standard formalism fails.

In an empiricist interpretation of the formalism we must be careful not to attribute the measurement result to the object. In the case of a nonideal photon measurement this is virtually self–evident: the number m of registered photons is derived from the output pulse of the photodetector and, as such, is related only very indirectly with the preparation. Things seem different in the case of an ideal detector. In that case it is tempting to interpret the number m as the number of photons that was present in the initial state. In a certain sense this may be correct (cf. sect. 5.). We must be very cautious on this issue, however, because it is at the very heart of QM. For this reason we shall entertain here an empiricist interpretation also in the ideal case. Then, the relation (10) should just be interpreted as a relation between two measurement procedures, one described by the PVM $\{N_n\}$, the other one by the POVM $\{M_m\}$.

3.2. Nonideal measurements

Generalizing (10) we now introduce the following definition of a nonideal measurement of an observable represented by the POVM $\{E_n\}$ (not necessarily a PVM):

The POVM $\{M_m\}$ represents a nonideal measurement of $\{E_n\}$ if a stochastic matrix (λ_{mn}) exists, such that

$$M_m = \sum_n \lambda_{mn} E_n, \tag{11}$$

$$\lambda_{mn} \geq 0, \sum_m \lambda_{mn} = 1. \tag{12}$$

The matrix (λ_{mn}) expresses the amount of nonideality involved in an interpretation of the $\{M_m\}$ measurement as a nonideal $\{E_n\}$ measurement. Note that the notion of "nonideal measurement" is a relative notion, relating two measurement procedures. Thus, if $(\lambda_{mn}) = (\delta_{mn})$ the $\{M_m\}$ measurement is an ideal $\{E_n\}$ measurement, even if the latter one is a nonideal measurement of some other observable. Note also that the relation (11) is *independent of the preparation*. The relation between the measurement probabilities of $\{M_m\}$ and $\{E_n\}$,

$$p_m = Tr\rho M_m = \sum_n \lambda_{mn} Tr\rho E_n, \tag{13}$$

on the contrary is determined by *both* the preparation and the measurement procedures.

As seen from (12) the quantities λ_{mn} can be interpreted as conditional probabilities. We should be careful, in the empiricist interpretation, not to interpret these quantities as measurement probabilities given that the object was in some state parametrized by n. A more appropriate interpretation is, to consider the quantity

$$p_{mn} = \lambda_{mn} Tr\rho E_n \tag{14}$$

as the joint probability distribution for the observables $\{M_m\}$ and $\{E_n\}$, the probability distributions of the latter observables being obtained as the marginals of (14).

Of course, the purpose of performing a nonideal measurement of $\{E_n\}$ is to obtain information on this latter observable that is as accurate as possible. Often it is impossible to attain complete accuracy. In that case the nonideal measurement is an inaccurate measurement. If, however, the nonideality matrix (λ_{mn}) has an inverse, then the relation (13) can be inverted, thus allowing to obtain *accurate* information on the probability distribution $\{Tr\rho E_n\}$ by means of a *nonideal* measurement of the observable $\{E_n\}$. If the inverse matrix (λ^{-1}_{nm}) of (λ_{mn}) exists, (11) can be inverted according to

$$E_n = \sum_m \lambda^{-1}_{nm} M_m. \tag{15}$$

Thus, the nonideal photon counting process (10) has such an inverse:

$$N_n = \sum_{m=n}^{\infty} \binom{m}{n} (-1)^{m-n} \xi^{-m} (1-\xi)^{m-n} M_m. \tag{16}$$

From (12) it immediately follows that

$$\sum_n \lambda^{-1}_{nm} = 1. \tag{17}$$

Note, however, that the matrix elements of the inverse matrix need not be nonnegative. Hence (15) does not generally satisfy the conditions (12) of a nonideal measurement. Invertibility will play an important role in our discussion of the Bell inequalities (cf. sect. 4.). In the present section we want to discuss further the idea of nonideality expressed by (11) and (12), and deriving from the possibility that the nonideal measurement yields a "wrong" answer if $(\lambda_{mn}) \neq (\delta_{mn})$. In Ref. 7 the relation (11, 12) was studied from the point of view of its ability of defining equivalence classes of measurement procedures. A *maximal* observable was then defined as an observable for which it is impossible to find an *inequivalent* observable that is measured by it in a nonideal way. Hence, the maximal observables correspond with measurements that in a certain sense are most ideal. It was demonstrated in Ref. 7 that in order to be a maximal observable the operators M_m must be proportional to projection operators on *one*–dimensional subspaces of Hilbert space. Hence, maximal PVM's corresponding with maximal orthogonal resolutions of the identity, are maximal observables. They are not the only ones, however. For instance, the POVM $\{M(d^2\alpha) = \frac{1}{\pi}\left|\alpha\right\rangle\left\langle\alpha\right| d^2\alpha\}$, corresponding with the overcomplete set of coherent states $\left|\alpha\right\rangle$ is also a maximal observable. It is interesting to note that from the point of view of the nonideality concept there is no difference between an orthogonal complete set of quantum states and a nonorthogonal overcomplete one. We will return in section 5. to the question of what meaning this similarity can have as regards the relation between the measurement results and possible properties of the object itself. Here it suffices to note that maximal observables cannot be improved on in the sense of the nonideality definition (11,12).

If the observable $\{E_n\}$ in (13) is a PVM it is possible to prepare the object corresponding to a density operator that makes the probability distribution $\{Tr\rho E_n\}$ dispersionless. In that case the probability distribution $\{p_m\}$ defined by (13) in general is dispersive (unless λ_{mn} differs from zero for one value of m only). In general the nonideal measurement seems to yield a probability distribution that is broader than the one obtained from a more ideal measurement. This points into a direction in which *two* contributions to the dispersion of the measurement outcomes of a nonideal measurement can be distinguished, viz., one stemming from preparation and one stemming from the nonideality of the measurement. The validity of this idea is strongly suggested by a proof that can be given if it is required that the nonideal measurement for any density operator yield the same expectation value as a more ideal measurement would do (this is often referred to as the requirement of "unbiasedness"). In that case it follows[(31)] that, if $\{E_n\}$ is a PVM, then the variances satisfy

$$\Delta^2(\{M_m\}) \geq \Delta^2(\{E_n\}). \tag{18}$$

Hence, if $\Delta(\{E_n\})$ could be attributed to the preparation, the nonideal measuring process would contribute the positive amount $\Delta(\{M_m\}) - \Delta(\{E_n\})$ to the standard deviation. Often measurement noise is treated along these lines.[(32)] Then (18) guarantees that two nonideal measurements of incompatible PVM's satisfy the Heisenberg inequalities because the PVM's do so.

The notion of unbiasedness hinges on the (eigen)values of the observables. In an empiricist interpretation of QM we do not want to rely on these (eigen)values (cf. sect. 2.2.). For this reason it would be inconsistent to rely on the notion of unbiasedness.[(33)] Also, this notion does not seem to have much physical relevance for the inefficient photodetector (10) which is clearly biased[6]. Then inequality (18) may be

[6]Often unbiasedness can be saved by attributing values to $\{M_m\}$ that are different from those of $\{E_n\}$.[(7)] Thus, for the photodetector with efficiency ξ we could choose ξm as the values of M_m. This, however, would be in conflict with an interpretation of the observable $\{M_m\}$ as describing a (nonideal) photon *counting* experiment.

violated, thus frustrating the possibility of proving the general validity of the Heisenberg inequalities for nonideal measurements. In particular, we have no reason to expect these to be satisfied if the two observables are measured nonideally in two separate experiments.

3.3. Joint nonideal measurement of incompatible observables

We shall now review an extension of the theory of nonideal measurements capable of dealing with the joint measurement of incompatible observables, and yielding an unambiguous description of mutual disturbance in such experiments, inherent in Bohr's idea of complementarity as illustrated by the "Gedanken" experiments.[7] Generalizing (5) we describe a joint measurement of two observables by a bivariate POVM $\{R_{mn}\}$, such that the joint probability distribution of the experiment is given by

$$p_{mn} = Tr\rho R_{mn}. \tag{19}$$

The two marginals $\{\sum_n R_{mn}\}$ and $\{\sum_m R_{mn}\}$ represent the two observables measured jointly. It was demonstrated[14, 34] that if these marginals are PVM's, these observables necessarily must be compatible. Hence, also in the generalized theory undisturbed joint measurement of incompatible PVM's is impossible. However, it is possible that each of the marginals represent a *nonideal* measurement of a PVM in the sense of definition (11,12):

$$\sum_n R_{mn} = \sum_k \lambda_{mk} E_k, \lambda_{mk} \geq 0, \sum_m \lambda_{mk} = 1, \tag{20}$$

$$\sum_m R_{mn} = \sum_l \mu_{nl} F_l, \mu_{nl} \geq 0, \sum_n \mu_{nl} = 1, \tag{21}$$

in which $\{E_k\}$ and $\{F_l\}$ may be incompatible PVM's.

In order to have a specific application of this formalism, let us consider the joint measurement of two incompatible components of photon polarization, an example first discussed by Busch.[26] The joint measurement is realized by means of a beam splitter (cf. Fig. 1), which either transmits the photon into the direction of the polarizer θ_1, or reflects it towards the polarizer θ_2. If the transmission probability of the beam splitter is γ, then the detection probabilities of detectors D_1 and D_2 are given by $\gamma Tr\rho E_m$ and $(1-\gamma)Tr\rho F_n$, respectively, m and n both having the two possible values "yes" and "no" corresponding with the two possible responses of the detectors[7]. The joint detection probabilities for the two detectors can be calculated simply from these probabilities because $p(yes, yes) = 0$. This immediately yields the bivariate POVM

$$(R_{mn}) = \begin{pmatrix} 0 & \gamma E_+ \\ (1-\gamma)F_+ & 1 - \gamma E_+ - (1-\gamma)F_+ \end{pmatrix}, \tag{22}$$

[7]It is also possible to include detector efficiency in the formalism. For simplicity this is omitted here.

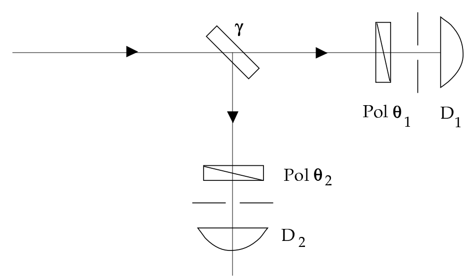

Figure 1: Joint nonideal measurement of two incompatible polarization observables

in which we have used the convention $yes=+, no=-$. Calculating the two marginals of (22) we find

$$\begin{pmatrix} \sum_n R_{+n} \\ \sum_n R_{-n} \end{pmatrix} = \begin{pmatrix} \gamma & 0 \\ 1-\gamma & 1 \end{pmatrix} \begin{pmatrix} E_+ \\ E_- \end{pmatrix}, \tag{23}$$

$$\begin{pmatrix} \sum_m R_{m+} \\ \sum_m R_{m-} \end{pmatrix} = \begin{pmatrix} 1-\gamma & 0 \\ \gamma & 1 \end{pmatrix} \begin{pmatrix} F_+ \\ F_- \end{pmatrix}, \tag{24}$$

defining the nonideality matrices (λ_{mk}) and (μ_{nl}) according to (20,21). It is easily verified that all requirements for the joint nonideal measurement of the two incompatible PVM's $\{E_m\}$ and $\{F_n\}$ are fulfilled.

By now quite a number of joint measurement procedures have been analyzed in an analogous way. Thus, it is possible to interpret certain neutron interference experiments as a joint measurement of interference and path[26] (an optical experiment performed by Mittelstaedt, Prieur, and Schieder[28] can be interpreted quite analogously). Optical detection techniques like eight-port homodyning and heterodyning of monochromatic light can be interpreted as joint measurements of the two quadrature components ("position" and "momentum").[26]

3.4. Complementarity

Bohr's complementarity idea is clearly illustrated by the two nonideality matrices (λ_{mk}) and (μ_{nl}) of (23,24). As γ varies from 0 to 1 the measurement changes from an ideal $\{F_n\}$ measurement to an ideal $\{E_m\}$ measurement, the other one simultaneously becoming more nonideal. For the extreme values of γ the measurement actually yields information about one observable only, the other marginal being reduced to the uninformative observable $\{O, I\}$. It seems to us that it is precisely this aspect that is demonstrated by the Bohr–Heisenberg "Gedanken" experiments. It must be emphasized that the nonideality matrices are determined completely by the POVM $\{R_{mn}\}$, and are independent of the initial state of the object system, i.e., the density operator. Hence, the complementarity as expressed by (23,24) is a property of the measurement procedure only, and is independent of the preparation. For this reason the nonideality embodied in these matrices must be considered to be caused by the measurement, and cannot be attributed to limitations of the quantum mechanical preparation that, according to Ballentine, can be held responsible for the Heisenberg inequalities (cf. sect. 2.3.). Contrary to Ballentine's belief(4) it is clear from sect. 3.3. that the quantum mechanical formalism *does* have to say something about the joint measurement of incompatible observables, and about the mutual disturbance of the incompatible observables measured jointly.

As we shall see now, however, Ballentine was right in stressing that the Heisenberg inequalities do not describe the limitation on joint measurement discussed here, because they refer to separate *ideal* measurements of PVM's. For this reason the Heisenberg inequalities are not related to the nonidealities as represented by the two nonideality matrices of (20,21). Yet, it can be demonstrated that the complementary behavior as exhibited by the example of (23,24) is a general property of joint nonideal measurements of incompatible PVM's, thus corroborating the Bohr–Heisenberg intuition as embodied in the "Gedanken" experiments. Restricting ourselves to maximal PVM's on an N–dimensional Hilbert space we can, in order to see this, make use of the average row entropy

$$\delta_{(\lambda)} = -\frac{1}{N}\sum_{m,k} \lambda_{mk} \log \frac{\lambda_{mk}}{\sum_{k'} \lambda_{mk'}} \tag{25}$$

as a nonideality measure for the matrix (λ_{mk}). It is easily demonstrated that this quantity is bounded by the values 0 (for an ideal measurement) and $\log N$ (in case of an uninformative measurement, for which $\lambda_{mk} = \frac{1}{N}$). It is important to note that the nonideality measure (25) does not depend on the (eigen)values of the observable. This makes this quantity particularly suitable in the context of an empiricist interpretation.

Defining an analogous measure for (μ_{nl}) it is possible(7) to derive the following inequality:

$$\delta_{(\lambda)} + \delta_{(\mu)} \geq \log max_{m,n} Tr E_m F_n. \tag{26}$$

If the two PVM's $\{E_m\}$ and $\{F_n\}$ have no eigenfunctions in common, then the right–hand side of this inequality is positive, thus yielding a nontrivial lower bound for the left–hand side.

Analogous relations can be derived if the two PVM's have eigenfunctions in common,(7) as well as for the joint nonideal measurement of observables like P and Q on infinite dimensional Hilbert space. Once again it is important to note that the inequality (26) is independent of the (eigen)values of the observables. Therefore the inequality holds independently of the requirement of unbiasedness.

Summarizing, it is seen that the quantum mechanical formalism actually contains two different types of inequalities, viz., i) inequalities like the Heisenberg inequality (3) that can be interpreted as restrictions on the possibility of *preparing* a quantum mechanical object; ii) inequalities like (26) by which *measurement procedures* are restricted. In both cases incompatibility of quantum mechanical quantities is at the basis of the pertinent restriction, thus demonstrating the common origin of both inequalities. However, the correspondence does not seem to go beyond this. Preparation and measurement are two quite different procedures using fundamentally different types of equipment, and described by different mathematical entities. For this reason it is not very surprising that they are governed by quite different types of relations.

In the description of the "Gedanken" experiments by Bohr and Heisenberg the different contributions of preparation and measurement to inaccuracy were not distinguished clearly enough. This essentially is the cause of Ballentine's criticism. Due to the extension of the mathematical formalism to positive operator–valued measures we are now able to make this distinction. It turns out[(35)] that the Bohr–Heisenberg uncertainty principle actually consists of two different principles, viz., i) a scatter principle, ii) an inaccuracy principle. The first one is represented by Heisenberg–type inequalities like (3), signifying that it is impossible to prepare the object in a quantum state such that these inequalities are violated even if the quantities are measured in the most ideal way. So, the quantities involved in (3) are attributed to the preparation procedure rather than to any disturbance caused by the measurement, even if these quantities are properties of probability distributions of *measurement results*. On the other hand inequalities like (26) are expressions of the excess inaccuracy that is inevitably introduced by the measurement procedure if this procedure aims at the joint (nonideal) measurement of two incompatible observables. This inaccuracy principle[8] can be interpreted as the implementation within the formalism of the "mutual exclusiveness" of the measurement arrangements of incompatible observables. It must be realized, however, that this "mutual exclusiveness" is not as rigorous as Bohr suggests. Interference is only destroyed completely in the two–slit experiment if path is measured ideally and, hence, the interference measurement is reduced to an uninformative one.[(26, 28)] In the intermediate situations in which *both* interference and path are measured nonideally, however, the nonideality matrices turn out to be both invertible, thus allowing the ideal probability distributions of both observables to be obtained from one and the same measurement arrangement.

3.5. Wigner measures

If the inverses of both nonideality matrices (λ_{mk}) and (μ_{nl}) exist, it is possible to calculate a *Wigner measure* $\{W_{kl}\}$ from the POVM $\{R_{mn}\}$ according to

$$W_{kl} = \sum_{mn} \lambda^{-1}_{km} \mu^{-1}_{ln} R_{mn}. \tag{27}$$

[8]Maybe "nonideality principle" would be a better name for this principle because in case of invertibility of the nonideality matrices there is, strictly speaking, no inaccuracy.

From (17) it follows that

$$\sum_{kl} W_{kl} = I, \tag{28}$$

characterizing $\{W_{kl}\}$ as a measure. Note, however, that unlike the operators R_{mn} of the POVM the operators W_{kl} need not be nonnegative. Hence, in general $\{W_{kl}\}$ is a quasi–measure. From (20,21) it also follows that

$$\sum_{l} W_{kl} = E_k, \tag{29}$$

$$\sum_{k} W_{kl} = F_l. \tag{30}$$

Hence, the ideal information on the observables $\{E_k\}$ and $\{F_l\}$ can be obtained from the Wigner measure.

For the POVM (22) we find

$$(\lambda^{-1}) = \begin{pmatrix} \gamma^{-1} & 0 \\ 1-\gamma^{-1} & 1 \end{pmatrix}, \quad (\mu^{-1}) = \begin{pmatrix} (1-\gamma)^{-1} & 0 \\ 1-(1-\gamma)^{-1} & 1 \end{pmatrix}, \tag{31}$$

from which the Wigner measure corresponding to the experiment of Fig. 1 is found according to

$$(W_{kl}) = \begin{pmatrix} 0 & E_+ \\ F_+ & E_- - F_+ \end{pmatrix}. \tag{32}$$

It is interesting to note that this Wigner measure is independent of the parameter γ of the measurement arrangement.

4. JOINT MEASUREMENT AND THE BELL INEQUALITIES

Unlike the EPR proposal (sect. 2.1.) in which it is essential that not all quantities that are involved in the reasoning are actually measured, in experimental tests of the Bell inequalities (e.g., Ref. 2) only *measured* quantities are essential. Actually, in the Bell inequalities four observables are involved that are not all mutually compatible. In the usual EPR experiments[9] only compatible pairs are measured jointly. In the original derivation by Bell[(36)] of the inequalities, a relation between incompatible observables was established by means of the assumption that the measurement results must be derivable from an underlying classical theory, a so–called hidden variables theory. In this section we want to demonstrate that our theory of joint measurement of incompatible observables makes it possible to discuss the role of the Bell inequalities in QM *without any such assumption.* Throughout the present section we will entertain the same empiricist interpretation that was used before. The role of hidden variables will be considered in later sections.

[9] Notwithstanding the fundamental difference between the experimental situations we shall stick to the common usage of referring to measurements testing the BI as “EPR experiments”.

Our treatment of the problem is based on the possibility of measuring jointly, be it nonideally, the four observables that are involved in the Bell inequalities. If a joint measurement arrangement exists for the four observables, then we obtain for every individual object that interacts with it a value for each of the four observables as a measurement result. By repeating the measurement we thus obtain a *quadrivariate* joint probability distribution (jpd). Then, a theorem due to Fine[37] and Rastall[38] (see also Ref. 39) directly tells us that the Bell inequalities must be satisfied because of the mere existence of such a quadrivariate jpd. Thus, denoting this jpd by p_{ijkl}, and its marginals by $p_{ij} = \sum_{kl} p_{ijkl}$, etc., the inequality

$$0 \leq p_{ik} - p_{il} + p_{jk} + p_{jl} - p_j - p_k \leq 1 \tag{33}$$

can straightforwardly be derived. This inequality was already derived for local hidden variables theories by Clauser and Horne.[40] It, however, can be derived without any reference to hidden variables if a quadrivariate jpd exists. From (33) it is possible to derive the inequality (81) as originally found by Bell,[36] for the correlation functions of the measurement results.[39] Since the correlation functions depend on the values of the measurement results, whereas the probabilities in (33) are independent of these values, we prefer in our empiricist approach to use this latter inequality. As is well–known this is no drawback at all since it are in general precisely the probability distributions obtaining in (33) that are compared with experimentally obtained ones.

4.1. Joint nonideal measurement of four observables

As discussed in sect. 3. a quantum mechanical measurement is described by a positive operator–valued measure. A joint nonideal measurement of four quantum mechanical observables can be represented by a generalization of (19) according to

$$p_{ijkl} = Tr\rho R_{ijkl}, \tag{34}$$

in which $\{R_{ijkl}\}$ is a quadrivariate POVM.

As in (20) and (21) it is required that the marginals $\sum_{jkl} R_{ijkl}$, etc., represent nonideal measurements of the four observables that are involved. In order to illustrate the feasibility of this scheme, we shall first deal with a generalization of the measurement arrangement of Fig. 1 yielding a generalization of the POVM (22) to four incompatible polarization observables as required by (34)(cf. Fig. 2). Instead of one beam splitter we now have three (transparancies $\gamma_1, \gamma_2, \gamma_3$). The incoming photon now has a certain probability to reach one of the detectors $D_i, i = 1, ..., 4$, each of these being positioned behind a polarizer in direction θ_i. Denoting the polarization observable corresponding with direction θ_i by $\{E^i_+, E^i_-\}, i = 1, ..., 4$, it is not difficult to find the quadrivariate jpd of this experiment according to

$$\begin{aligned} p_{+---} &= \gamma_1\gamma_2 Tr\rho E^1_+, & (35)\\ p_{-+--} &= \gamma_1(1-\gamma_2) Tr\rho E^2_+, & (36) \end{aligned}$$

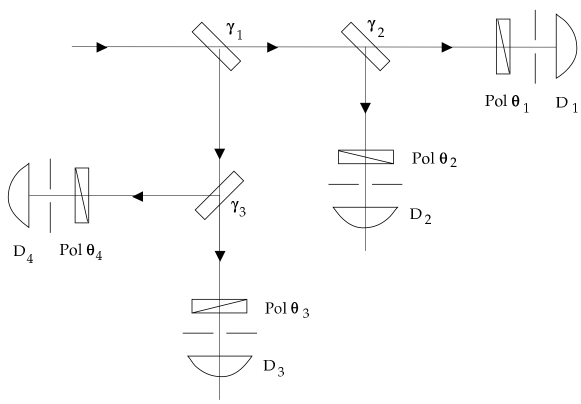

Figure 2: Joint nonideal measurement of four incompatible polarization observables

$$
\begin{aligned}
p_{--+-} &= (1-\gamma_1)\gamma_3 Tr\rho E^3_+, && (37)\\
p_{---+} &= (1-\gamma_1)(1-\gamma_3) Tr\rho E^4_+, && (38)\\
p_{----} &= 1-(p_{+---}+p_{-+--}+p_{--+-}+p_{---+}), && (39)
\end{aligned}
$$

all other p_{ijkl} vanishing. This defines the quadrivariate POVM $\{R_{ijkl}\}$ as

$$
\begin{aligned}
R_{+---} &= \gamma_1\gamma_2 E^1_+, && (40)\\
R_{-+--} &= \gamma_1(1-\gamma_2) E^2_+, && (41)\\
R_{--+-} &= (1-\gamma_1)\gamma_3 E^3_+, && (42)\\
R_{---+} &= (1-\gamma_1)(1-\gamma_3) E^4_+, && (43)\\
R_{----} &= 1-(R_{+---}+R_{-+--}+R_{--+-}+R_{---+}), && (44)
\end{aligned}
$$

all other R_{ijkl} vanishing. It can easily be verified that the marginals $\sum_{jkl} R_{ijkl}$, etc., of this POVM indeed represent nonideal measurements of the four polarization observables. Analogously to (23,24) we obtain

$$
\begin{pmatrix} \sum_{jkl} R_{+jkl} \\ \sum_{jkl} R_{-jkl} \end{pmatrix} = \begin{pmatrix} \gamma_1\gamma_2 & 0 \\ 1-\gamma_1\gamma_2 & 1 \end{pmatrix} \begin{pmatrix} E^1_+ \\ E^1_- \end{pmatrix}, \quad (45)
$$

$$
\begin{pmatrix} \sum_{ikl} R_{i+kl} \\ \sum_{ikl} R_{i-kl} \end{pmatrix} = \begin{pmatrix} \gamma_1(1-\gamma_2) & 0 \\ 1-\gamma_1(1-\gamma_2) & 1 \end{pmatrix} \begin{pmatrix} E^2_+ \\ E^2_- \end{pmatrix}, \quad (46)
$$

$$
\begin{pmatrix} \sum_{ijl} R_{ij+l} \\ \sum_{ijl} R_{ij-l} \end{pmatrix} = \begin{pmatrix} (1-\gamma_1)\gamma_3 & 0 \\ 1-(1-\gamma_1)\gamma_3 & 1 \end{pmatrix} \begin{pmatrix} E^3_+ \\ E^3_- \end{pmatrix}, \quad (47)
$$

$$\begin{pmatrix} \sum_{ijk} R_{ijk+} \\ \sum_{ijk} R_{ijk-} \end{pmatrix} = \begin{pmatrix} (1-\gamma_1)(1-\gamma_3) & 0 \\ 1-(1-\gamma_1)(1-\gamma_3) & 1 \end{pmatrix} \begin{pmatrix} E^4_+ \\ E^4_- \end{pmatrix}. \tag{48}$$

For $0 < \gamma_i < 1$ all nonideality matrices in (45) through (48) are invertible. Hence this measurement allows the joint determination of the probability distributions of all PVM's $\{E^i_+, E^i_-\}, i = 1, \ldots, 4$. Denoting the nonideality matrices in (45) through (48) by $(\lambda^i), i = 1, \ldots, 4$, we may define for the present experiment, analogously to (27), the Wigner measure

$$W_{pqrs} = \sum_{ijkl} \lambda^{1^{-1}}_{pi} \lambda^{2^{-1}}_{qj} \lambda^{3^{-1}}_{rk} \lambda^{4^{-1}}_{sl} R_{ijkl}. \tag{49}$$

It is interesting to note that we may choose the polarizer directions θ_i such that all four PVM's are mutually incompatible. For the POVM (40-44) we obtain from (49):

$$W_{+---} = E^1_+, \tag{50}$$
$$W_{-+--} = E^2_+, \tag{51}$$
$$W_{--+-} = E^3_+, \tag{52}$$
$$W_{---+} = E^4_+, \tag{53}$$
$$W_{----} = 1 - (W_{+---} + W_{-+--} + W_{--+-} + W_{---+}), \tag{54}$$

all other W_{pqrs} vanishing. This has the important consequence that this measurement is a *complete* measurement[41]: from the experimental probabilities (35) through (39) it is possible to obtain a complete determination of the quantum mechanical polarization state of the incoming photon.

Because of the existence of the quadrivariate jpd (34) the measurement results of this experiment do satisfy the Bell inequalities (BI). It seems to us that this must be a consequence of the existence of a measurement setup for the joint (nonideal) determination of the four observables that are involved. Due to this fact every incoming photon yields a value for each of the four polarization observables. Because of the incompatibility of the observables there is a mutual disturbance as discussed in section 3., causing the measurement results to differ from the ideally measured ones. In our opinion this strongly suggests that deviations from the BI must be related to the mutual exclusiveness of measurement arrangements of incompatible observables, which may be responsible for the restricted applicability of counterfactual definiteness in QM (cf. sect. 7.): the correlation between the values of incompatible observables evidently depends on the way these observables are measured jointly.

Often the difference between experiments satisfying and not satisfying the BI is discussed in terms of "classical" versus "quantum" correlations. Indeed, this difference can be traced back to the fact that the first ones are described by a POVM, its positivity warranting "classical" correlations, whereas the second ones are described by a Wigner measure: "quantum" correlations are related to negative probabilities. Yet, we should be careful in making such a distinction. As is seen from the present example the measurement results, even though they are "classically" correlated, can yield complete information on the quantum mechanical state, and, hence, also contain information on the "quantum" correlations. As a matter of fact, as demonstrated by inequality

(26) given in sect. 3., the mutual disturbance that is responsible for the "classical" appearance of the correlations of the joint measurement results satisfies a *quantum mechanical* restriction. For this reason it may be not completely warranted to say that within the well–defined context of a concrete experiment "everything is classical".

4.2. Measurements of EPR correlations

As a second example we consider an EPR–like experiment (cf. Fig. 3) in which a correlated two–photon system is prepared as is done, e.g., in the experiments by Aspect et al.[2, 3] In this experiment[10] two incompatible components of the polarization vector of each of the two photons are measured jointly in the sense discussed before (see also Ref. 42). Once again this is a joint (nonideal) measurement of four observables. In this experiment these four observables coincide with the observables that are usually measured two by two in EPR experiments performed for testing the Bell inequalities. However, even if we choose both the conditions of preparation and the polarization directions such that the BI are violated in these latter tests, yet in the present experiment we find the BI to be satisfied. This must be so, because in the joint measurement of the *four* observables we once again have a quadrivariate jpd. The quadrivariate POVM of this jpd is found easily as

$$R_{m_1 n_1 m_2 n_2} = R^1_{m_1 n_1} R^2_{m_2 n_2}, \tag{55}$$

$R^i_{m_i n_i}, i = 1, 2$ being given by (22) for the two arms of the experimental arrangement.

In the usual EPR experiments the jpd's $Tr\rho E^1_{k_1} E^2_{k_2}$, etc. are measured. As before also here these probability distributions can be calculated from a Wigner measure:

$$W_{k_1 l_1 k_2 l_2} = W^1_{k_1 l_1} W^2_{k_2 l_2}, \tag{56}$$

$W^i_{k_i l_i}$ for each arm $i = 1, 2$ being given by (32).

It is interesting to note that the transition from the "classical" correlations satisfying the BI and described by the POVM (55), to the "quantum" correlations described by the Wigner measure (56) and (possibly) violating the BI, is realized by two separate transitions

$$R^i_{m_i n_i} \to W^i_{k_i l_i}, i = 1, 2. \tag{57}$$

In view of our earlier interpretation it would be natural to interpret each separate transition as a cancellation of the mutual disturbance of the measurement results in a joint measurement of two incompatible observables in each arm of the interferometer. Violation of the BI is often attributed to nonlocal influences from one arm of the

[10]If we compare this experiment with Aspect's switching experiment,[3] the two switching elements are now replaced by *static* semitransparant mirrors, directing the photon to either one of the two polarizers behind each mirror. In Ref. 3 the quantities γ_1 and γ_2 are intended to be random variables that can take either the values 0 or 1.

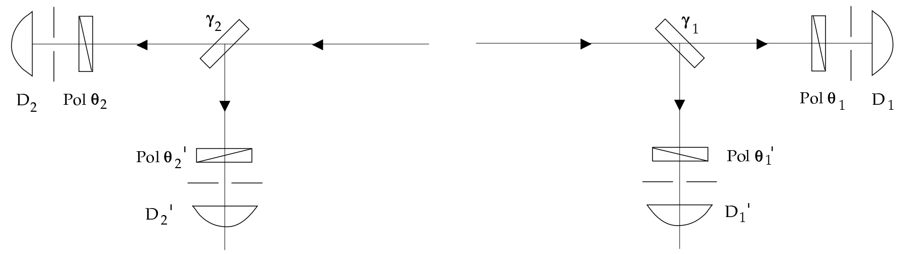

Figure 3: EPR-like joint nonideal measurement of four polarization observables

interferometer on the measurement that is performed in the other arm. It must be stressed here that nothing in the formalism presented above warrants such a conclusion of nonlocal influences. Both (55) (which satisfies the BI) and (56) (which may violate the BI) has the form of a *product* of operator–valued measures pertaining to the two photons. Hence, if the state of the two–photon system is a product state (because of independent *preparation* of the two photons) also the *measurement results* in one arm of the interferometer are independent of those found in the other arm, *both* for (55) and (56). Also, even if the preparation is in a *correlated* two–photon state the marginals $R^i_{m_in_i}, i = 1,2$ and $W^i_{k_il_i}, i = 1,2$ describing the measurement results in one arm, are independent of the presence of any measuring apparatus in the other arm. Evidently this is all completely independent of whether the Bell inequalities are violated or satisfied. If nonlocal influences were responsible for violation of the BI we might also wonder why it is impossible that these influences are unable to cause a violation of the BI for the measurement results in measurement arrangements like the one of Fig. 3.

On the other hand, an interpretation in terms of a disturbance that is due to the joint measurement of incompatible observables seems to reduce the problem of the Bell inequalities to the feature that is most distinguishing between classical and QM, viz., the existence of *incompatible* observables. The Bell inequalities can be violated because of the mutual exclusiveness of measurement arrangements of *incompatible* observables rather than because of an influence exerted by the measurement of a *compatible* observable in the other arm of the interferometer.

In an empiricist interpretation of QM, in which the measurement result is interpreted as a property of the measuring instrument, it is very natural to accept the impossibility of attributing simultaneously values of incompatible observables to an object if no measurement arrangement exists that would allow to do this. In this interpretation it is possible, though, to compare different ways of jointly measuring incompatible observables. Then, the impossibility of attributing the same pair of measurement results to two incompatible observables measured jointly in two different ways, is naturally explained by the different ways in which the incompatible measurements mutually disturb each other. For this reason it does not seem to make much sense to combine measurement results obtained in *different* EPR measurements, as is done, e.g., in Refs. 43 and 23 (see also sect. 7.). Thus, comparing the two EPR measurements measuring observables $\{E^1_{k_1} E^2_{k_2}\}$ and $\{E^1_{k_1} F^2_{l_2}\}$, respectively, these can be interpreted as ideal measurements of $\{E^2_{k_2}\}$ and $\{F^2_{l_2}\}$, respectively. Such ideal measurements can, however, also be seen as two different joint nonideal measurements of $\{E^2_{k_2}\}$ and $\{F^2_{l_2}\}$ in which $\{F^2_{l_2}\}$ is maximally disturbed if $\{E^2_{k_2}\}$ is measured, and vice versa. Actually, if we choose $\gamma = 1$ in (22) the POVM reduces to a POVM having the PVM $\{E_+, E_-\}$ as one marginal and the uninformative $\{O, I\}$ as the other one, indicating that in the limit in which $\{E_+, E_-\}$ is measured ideally all information on $\{F_+, F_-\}$ is wiped out. If we choose $\gamma = 0$ in (22) the same holds true if $\{E_+, E_-\}$ and $\{F_+, F_-\}$ are interchanged. Hence, we have no single reason to suppose the correlations between $\{E^2_{k_2}\}$ and $\{F^2_{l_2}\}$ to be the same in these two experiments, even if conditioned on the same measurement outcome for the observable $\{E^1_{k_1}\}$ measured jointly with both of them.

By taking in (55) $(\gamma_1, \gamma_2) = (0,0), (0,1), (1,0)$ or $(1,1)$, respectively, we obtain the four EPR measurements that are usually considered in relation to the Bell inequalities. Hence, these can be interpreted as four different joint nonideal measurements of the four PVM's that are involved. In this way four quadrivariate POVM's are obtained, indicating that in each of the four EPR experiments the BI must be satisfied. Note, however, that in such EPR experiments no information is obtained on the other (incompatible) observables. For this reason there is no empirical verification of the BI in such an experiment. Usually the BI (33) are tested by combining bivariate probabilities obtained in *different* experiments. These bivariate jpd's can be derived from *different* quadrivariate jpd's, obtained from (55) by inserting the different extreme values of (γ_1, γ_2) mentioned above[11] Since in the derivation of (33) it was essential that the bivariate jpd's are derived from the *same* quadrivariate jpd we have, in the context of the theory of joint nonideal measurement, no reason to surmise that there is any strangeness in the fact that certain combinations of EPR experiments do not satisfy (33). We just can draw the conclusion that, if the BI are not satisfied, there cannot be a common quadrivariate jpd for all four EPR experiments, or, that these, interpreted as joint nonideal measurements of all four PVM's that are involved, must be *differently disturbing*.

It seems to us that the two experiments discussed here (Fig. 2 and Fig. 3) are conceptually not different. The main difference is that in the first example the measurements of *all* observables disturb each other, whereas in the second one some of the observables are mutually compatible. Such compatibility has, during a long time, been necessary in order that the Bell inequalities could be put to an experimental test, because only the joint measurement of *compatible* observables was thought to be possible. It seems to us that this restriction has considerably confused the issue by directing the attention more to the possible (nonlocal) disturbance of one observable

[11]For instance, $p_{+-+-} = 0$ for all ρ if $(\gamma_1, \gamma_2) = (1,0), (0,1)$, or $(0,0)$, but equals $Tr\rho E^1_+ E^2_+$ for $(\gamma_1, \gamma_2) = (1,1)$, the latter being in general different from zero.

by the joint distant measurement on another object of another *compatible* observable, and by neglecting the possible effects due to the *incompatibility* of the observables that are involved. The possibility of measuring jointly, be it nonideally, incompatible observables makes it possible to free ourselves from the restriction mentioned above, and to study the relation between "classical" and "quantum" correlations directly by measuring all observables jointly. In this context experiments involving a number of compatible observables seem to be just special cases, without any experimental evidence that nonlocal interactions would be involved. On the contrary, all experimental results seem to be understandable perfectly well on the basis of a *local* interaction of object and measuring instrument.

5. QUANTUM MECHANICAL OBSERVABLES AND REALITY

5.1. Empiricism versus contextualistic realism

As discussed in sect. 2.2. there does not exist an unanimous opinion about the physical significance of the quantum mechanical formalism. Opinions range from QM as a *complete* description of the whole of reality, to QM as merely a symbolic formalism for predicting measurement outcomes, the latter being properties of a measuring instrument[12]. We prefer to employ the latter interpretation which is by far the weakest one, and, for this reason, least liable to paradoxical consequences. In this section we want to discuss in a more detailed way the problems encountered in a realist interpretation of the quantum mechanical formalism in which a result of a measurement is considered as a property of the microscopic object.

In discussing realist interpretations of the quantum formalism we must distinguish between two kinds of realism, viz., realism with respect to the wave function and realism with respect to observables. It is necessary to make such a distinction because it is possible to acknowledge the reality of measurement results (i.e., observables as properties of the microscopic object) without being obliged to attribute a realistic meaning to the wave function or density operator. One major example of this is the position of Bohr, discussed in sect. 2.1.. Precisely because of this fact Bohr's realism[(17)] does not lead to the usual paradoxes that are mostly stemming from a realist understanding of the wave function. Yet, in the present section we want to draw the attention to a certain weakness even of this moderate kind of realism, thus explaining our preference for a fully empiricist interpretation.

Do we have any experimental indications that it does make sense to attribute quantum mechanical observables as properties to quantum mechanical objects? Strictly speaking we have not, because the only data that are accessible to experimental verification within the domain of quantum mechanics are pointer readings of measuring instruments. For this reason the practice, already going back to Bohr and Heisenberg, of interpreting the result of a quantum mechanical measurement as a property

[12]In general the reality of *directly* observed phenomena like the macroscopic pointers of measuring instruments is taken for granted even in an empiricist interpretation of quantum mechanics. We shall adapt ourselves to this custom.

of the object system (as this property is in the context of the measurement), goes beyond an interpretation of QM that is minimally required by experimental evidence. On the other hand, certain experimental results would be rather puzzling if they were not based on some property of the object system. For instance, we can measure momentum of a free particle at two consecutive instants. Since momentum is conserved this can be described as a joint measurement of the two compatible observables $p(t_1) = p$ and $p(t_2) = p$ in the initial state ρ of the system, yielding the jpd $P(p,p') = Tr\rho\,|p\rangle\langle p|\,p'\rangle\langle\, p'\,|{=}\langle\, p'\,|\rho|\,p\,\rangle\rangle\,\delta(p-p')$. Without the assumption that to the object a well–defined value of p can be attributed in some sense as an objective property, it would be quite incomprehensible why this value must be the same for momentum measurements performed at different instants on the same particle[13]. On the other hand, if the object would have such an objective property that is conserved in time, the strict correlation between the measurement results would seem to be a natural consequence. As a second example we mention EPR experiments in which the correlation is measured between observables pertaining to two different particles while these are far apart after having interacted in the past. If the correlation found in the measurement results cannot be explained by a correlation of some properties of the particles, created in the interaction between the particles when these were near each other, we are bound to resort to such notions as nonlocality or inseparability[(46)] in order to be able to explain how the experimental correlations in EPR experiments come about. By Mittelstaedt the attribution of an observable to the object is indicated as an "objectification". [(47, 48)]

One major problem in attributing observables as properties to the object is the non–Boolean structure of QM which is due to the incompatibility of quantum mechanical observables. It is impossible in general to attribute simultaneously values to incompatible observables corresponding with noncommuting hermitian operators, in a way that is analogous to the attribution of properties in classical mechanics.[(47)] Such a simultaneous attribution would also cause the Bell inequalities to be satisfied because it would imply the existence of a quadrivariate jpd (cf. sect. 4.). In attributing simultaneously values to incompatible observables it seems that deviations must be expected from the ideal quantum mechanical measurement results. In sect. 3. these deviations were related to a mutual disturbance by the measurement procedures of incompatible observables, thus stressing the importance of the role of the measurement setup in the interpretation of quantum mechanics.

If QM would be interpreted as an *objective* description of physical reality, valid independently of the measurement arrangement necessary for measuring the observable, then paradoxes would arise, as already indicated by the EPR problem discussed in sect. 2.1.. An *objective* realist interpretation of quantum mechanics seems to be impossible, i.e., QM does not seem to describe an *objective* reality. To many physicists the problems discussed here have been a reason for a complete rejection of the idea of attributing quantum mechanical observables to the object, and accept a purely em-

[13]Such a correlation between measurement results of measurements performed on the same particle would seem to be a problem for both Heisenberg's interpretation of quantum mechanical probability in terms of actualization of a potentiality[(44)], and for Popper's closely related propensity interpretation.[(45)]

piricist interpretation of QM. Others restrict, like Bohr did, this attribution to those physical situations in which the object is interacting with the measuring instrument. In both views the quantum mechanical measurement result is thought to be created only in the act of measurement, either as a property of the measuring instrument or as a property of the object itself. In both cases a quantum mechanical observable can have no value unless this observable is actually measured. Then the impossibility of a simultaneous attribution of values to incompatible observables can be accounted for by the impossibility of having a measurement arrangement yielding simultaneously undisturbed values of incompatible observables, i.e., by the measurement context. Hence, as far as a realist interpretation of QM is possible, it seems that it must be a *contextualistic* realist one (cf. sect. 2.2.). It does not seem allowed to assume the possibility of attributing a result of a quantum mechanical measurement to the object as a property already possessed *before* the measurement. As far as QM describes reality it seems to be describing at best an *observed* reality that is interacting with a quantum mechanical measuring instrument. The value of a quantum mechanical observable is well–defined only within the context of the actual measurement of this observable.

5.2. EPR and contextualistic realism

It is interesting to note that the ambiguity observed by Bohr[9] in the EPR element of physical reality[8] precisely had its origin in the assumption that this element was to be an *objective* property of the object. According to Bohr this could not be the case, even in a measurement context in which no actual measurement is performed on this object (particle 2, cf. sect. 2.1.) but in which only a *correlated* particle (particle 1) is directly interacting with a measuring instrument. Evidently it is assumed by Bohr that the measurement context of particle 1 can serve also for defining a property of particle 2. One could presumably try to justify such an assumption by referring to the possibility of obtaining measurement results for *both* particles from the measurement on particle 1 alone. This possibility seems to extend the measurement context of particle 1 to both particles, thus suggesting the possibility of a contextualistic realist interpretation of a quantity of particle 2. Then the ambiguity observed by Bohr refers to the impossibility of having the same element of physical reality for particle 2 in the different contexts of incompatible observables like x_1 and p_1.

Such a justification, however, would not be completely convincing (see also Ref. 16). The reason for this judgment is that the reasoning strongly hinges on the *correlation* between the two particles. Only because of this correlation it is possible to infer a measurement result for particle 2 by performing a measurement on particle 1. In the reasoning this correlation is treated as an objective property of the particle pair, that is true independently of its experimental verification. On the other hand, however, with the correlation of the two observables, $\{E^1_{k_1}\}$ and $\{E^2_{k_2}\}$, a quantum mechanical observable (viz., $\{E^1_{k_1}E^2_{k_2}\}$) can be associated. Hence, this correlation is a quantum mechanical observable just like p_1 is. Therefore in a contextualistic realist interpretation of QM it has a value *only* in the context of a *correlation* measurement. For this reason it does not seem consistent in such an interpretation to assume the existence of a well–defined correlation between p_1 and p_2 if not *also* a momentum measurement on particle 2 is performed.

It seems to us that by not recognizing correlation as a quantum mechanical observable Bohr followed Einstein on the way of attributing quantum mechanical quantities to the object *as objective properties of the object* quite a bit further than can be justified on the basis of a contextualistic realist interpretation of observables. Much of the confusion caused by Bohr's answer[9] to the EPR paper[8] could have been prevented if Bohr had maintained his interactionalist interpretation more strictly and had referred more explicitly to the necessity of measuring the observables of *both* particles in order that QM be applicable to the EPR "Gedanken" experiment.

5.3. Contextualistic realism and first kind measurements

A contextualistic realist interpretation of observables in the sense discussed above might appear to be a rather innocent strengthening of the interpretation of the quantum formalism compared with the empiricist one. Indeed, it has gained wide acceptance and is responsible for Dirac's[49] idea that a second measurement of the same observable, performed immediately after the first one, will yield the same value of the observable. Yet, it seems to us that this strengthening of the interpretation has important consequences, and must be considered with a certain suspicion. As a matter of fact, consecutive measurements can be carried out, and Dirac's idea can be tested experimentally. Then it is not difficult to find examples of quantum mechanical measurements *not* satisfying it. For instance, in a proportional counter measuring the energy of a charged particle, the measurement outcome is determined by the amount of ionization effected by the particle. In an ideal experiment in which the height of the electric pulse (the pointer reading) is a fair measure of the energy of the incoming particle, the latter must lose all its kinetic energy in the ionization process. Hence, its final energy must be quite different from its initial one, thus preventing the pointer reading from being an indication of this final energy. Moreover, this is not a contingent matter, but it is basic to the principle of this measurement procedure. Hence, at least quantum mechanical measurement procedures exist that can hardly be reconciled within a contextualistic realist interpretation of observables. It is possible, though, to find measurement procedures that do satisfy Dirac's idea. For instance, a Stern–Gerlach measurement measuring one component of spin can to a good approximation be considered as such. Such measurement procedures are often called "measurements of the first kind". Such behavior, however, is the exception rather than the rule. In most quantum mechanical measurement procedures the state of the object will be changed by the first measurement so as to yield a *different* measurement result in a subsequent measurement of the same observable.

A (contextualistic) realism of the kind discussed above may be responsible for the fact that many physicists consider the measurement of the first kind as the paradigm of a quantum measurement. As is well known this is further implemented through the so–called "projection" or "reduction" postulate, telling us how the state of the object is changed by the first measurement so as to comply with Dirac's idea. From the foregoing example it will be clear that in *actual* quantum measurements the projection postulate cannot always be satisfied, at least not in a way matching Dirac's idea. It seems to us that a contextualistic realist understanding of QM has the drawback of restricting the class of quantum mechanical measurements to a subclass (the measurements of the first kind) that is hardly representative of measurements performed in actual practice. By doing so the danger exists that properties of the members of this subclass are presented as *generic* properties of quantum measurement, thus creating a certain alienation between theory and practice. Since measurement procedures like energy measurement by means of a proportional counter must be accepted as valid quantum measurement procedures, it is necessary to make a clear distinction between the property of the measuring instrument (the final pointer position) that is to be interpreted as corresponding with the measurement outcome, and the final state c.q. final properties of the object. Therefore the contextualistic realist interpretation of quantum mechanical observables does not seem acceptable in general, although in special cases it is not at variance with experimental data.

5.4. Contextualistic realism and Heisenberg's uncertainty principle

There is still another weakness to a contextualistic realist interpretation of observables: it does not solve the problem for which realism was taken recourse to, viz., the explanation of correlations of measurement results, mentioned before, in terms of *objective* properties of the object system, i.e., properties the object has independently of any measurement. With respect to "explanation" it does not perform any better than an empiricist interpretation in which the observable is thought to correspond with a property of the measuring instrument. For this reason a contextualistic realist interpretation cannot be the final answer to the problems of QM.

In discussions by Heisenberg and Bohr on the so–called "Gedanken" experiments a contextualistic realist terminology is employed in order to describe what is going on in quantum mechanical measurements. Thus, in the context of a position measurement momentum (seen as a property of the particle) would be disturbed. According to Bohr it would be possible to attribute a value to both position and momentum, be it with uncertainties Δq and Δp satisfying Heisenberg's uncertainty relation (3). If $\Delta q = 0$ this is generally interpreted as the object *having* a precise value of position, momentum being undefined completely in accordance with Heisenberg's relation.

It is not unimportant to note that Bohr and Heisenberg seem to have different opinions about the precise meaning of this. For Bohr it means that in the context of a position measurement momentum is not defined at all. So, it is impossible to attribute a value to momentum in the context of a position measurement. Heisenberg seems to think much more classically. Thus, it seems that with Heisenberg every particle has a well–defined value of momentum in a position measurement, be it that this value cannot be predicted with any finite precision due to the unpredictability of momentum disturbance by the position measurement. We shall refer to this as Heisenberg's disturbance theory of measurement. This difference between Bohr and Heisenberg can be extended to the case of joint nonideal measurement of position and momentum, inequality (3) having for Bohr the significance of a limitation on the possibility of *definition* of position and momentum for each individual particle, whereas for Heisenberg it is a lower bound on the *disturbance* of the object by the measurement, it remaining possible to attribute a well–defined value (be it a disturbed one, the precise values being predictable only to an extent limited by the Heisenberg inequalities) to both position and momentum of each individual particle.

In assessing the meaning of the "Gedanken" experiments insight has severely been hampered by the fact, alluded at in sect. 2.3.2, that only extreme cases were considered in which one of the uncertainties in (3) is infinite. The theory of joint nonideal measurement developed in sect. 3. makes it possible to consider more general cases, and to provide additional tests for the possibility of a contextualistic realism as implied by Heisenberg's disturbance theory of measurement. Let us first consider the nonideal photodetector described by the POVM (10). A contextualistic realist interpretation of the description of this measurement would boil down to an attribution to the electromagnetic field of a photon number corresponding to the number m of registered photons. According to this interpretation the uncertainty in the photon number induced by the measurement would be related by a Heisenberg inequality to a concomittant uncertainty in a complementary phase observable, the latter being caused by a disturbance of phase by the (nonideal) photon number measurement.

By itself, such an interpretation is not impossible, and could be tested by means of separate ideal measurements of photon number and phase subsequent to the nonideal photon number measurement that was considered. However, even if such a test would corroborate the contextualistic realist interpretation, it would seem that, as in the ideal measurements discussed before, this would be only a contingent property of the measurement procedure, *not* a general requirement to be imposed on a generic (nonideal) measurement procedure of photon number. Measurements are in general performed in order to obtain knowledge about the microscopic object, preferably knowledge that is as faithful as possible to a reality of the object that is undisturbed by the measurement. Thus, it is the *determinative* aspect of measurement that is above all of importance. The contextualistic realist interpretation, however, hinges on the *preparative* aspect of measurement rather than the determinative one. What is of importance, is that a photon number measuring instrument registers as many photons as possible of the photons that are present in the initial state. In which state the electromagnetic field is left by the measurement is of secondary importance (unless the measurement is a preparative one, as e.g., in quantum nondemolition measurements). In general in a photon counting experiment the photons that are registered are absorbed by the detector, and hence do not exist in the final state of the field. Hence, in a nonideal measurement the final state must contain the photons that are *not* registered. This is completely at variance with a contextualistic realist picture in which the measurement result is attributed (in the context of measurement) as a property to the object. For this reason a contextualistic realist interpretation of nonideal quantum measurements cannot be generally applicable. On the other hand, an empiricist interpretation of nonideal photon number measurements can make a distinction between the final pointer position indicating the number of registered photons and the number of photons that would be registered in an ideal measurement of photon number, the statistics of the relation between these measurements being described by (10).

A word of caution is in order here. Because of the possibility of a classical statistical explanation of the binomial relation (10) in terms of the detection probability of real photons, it is very tempting to complement the empiricist interpretation with a realistic picture of real photons being objectively present in the initial state, only a statistical fraction being registered. Like in the case of the "classical" appearance of quantum correlations discussed in sect. 4. it might, however, be preferable to resist this temptation, since this would imply an objectivistic realist interpretation of *ideal* measurement results. Such an interpretation would inevitably revive the problem of counterfactual definiteness to which we shall return in later sections.

As a further example consider the joint measurement of two incompatible polarization observables $\{E_m\}$ and $\{F_n\}$ (cf. Fig. 1). A disturbance interpretation in the Heisenberg sense would amount to an analysis of the experiment in which the $\{F_n\}$ observable is disturbed by the presence of that part of the measurement arrangement that is meant to measure $\{E_m\}$, thus causing an uncertainty in the value of the $\{F_n\}$ observable, and vice versa. In an empiricist interpretation this does not pose any problem because there are no further constraints on the association of pointer readings and values of the observables. In a contextualistic realist interpretation this may be different. Thus, each polarizer–detector pair in Fig. 1 is by itself, in principle, an *ideal* measuring device for the corresponding polarization observable, and can be interpreted to register the corresponding property of the photon as it is in interaction with this part of the measurement setup. Hence, an analysis along contextualistic realist lines would rather yield a well–defined value of either one of the two observables, depending on the path an individual photon has chosen in the semitransparant mirror, than values of both observables satisfying Heisenberg's inequalities in either the sense of Bohr or Heisenberg. Once again it is not clear how the experimental context could be used to attribute simultaneously, even in an "indeterminate" sense, values of both $\{E_m\}$ and $\{F_n\}$ as properties to each individual photon.

5.5. Unsharp reality?

By Busch[50] and Mittelstaedt[47] the joint measurement of incompatible observables is associated with the idea of "unsharp" reality, and presented as a formalization of Heisenberg's ideas about indeterminacy. In agreement with the conclusion reached in sect. 5.1. a realist interpretation of un "unsharp" observable could hardly be an *objective* realist one, because in the definition of the POVM the measurement procedure is essentially involved. An attempt in this direction would boil down to attributing to the object the objective property (the "value" of the unsharp observable) of having a certain probability for each measurement outcome *if the measurement of the unsharp observable is carried out.* Since it is also possible to perform a *sharp* measurement of the observable, this would imply a certain inconsistency to the effect that certain preparations (viz., the ones corresponding with eigenstates) would have simultaneously a sharp and an unsharp value of the same observable. For this reason it would seem that such a realist interpretation, if feasible at all, must be a contextualistic one, as also suggested by Mittelstaedt's reference to the measuring process in the semantics of quantum language, as well as by the dependence of the nonideality matrix (λ_{mn}) in (11), defining the relation between the "sharp" and the "unsharp" observable, on the measurement procedure. A contextualistic position is also taken by Ali and Prugovecki[51] who associate their "fuzzy phase space" to a system that is *not* "in isolation from the methods and instruments used in performing measurements on the system".

From the considerations of this section it will be clear that we do not find this *contextualistic* realist interpretation very attractive. The interpretation seems to burden us with ambiguities like the one observed with respect to indeterminacy in the joint polarization measurement, without providing explanation going beyond the one given by the empiricist interpretation[14]. It seems to us to be more preferable associating the notion of an "unsharp observable" like the nonideal photon number observable (10) with a nonideal (or unsharp or inaccurate) measurement of a reality that is prepared independently of which measurement is going to be performed. After each preparation of the object we can choose whether we shall perform either an ideal or a nonideal measurement of some observable $\{E_n\}$. The relation (11) between the two POVM's then describes in a natural way the amount of information that is lost in the nonideal measurement as compared with the ideal one. It is not so much reality that is unsharp in an "unsharp" measurement; it is rather our way of obtaining information on reality that introduces unsharpness in our observations. If we would perform on the free particle discussed earlier a *nonideal* momentum measurement subsequent to an ideal one, we would not find the strict correlation that would be found if both measurements were ideal ones. These different results must in our opinion be attributed to a different functioning of the two measurement procedures, the state of the object prior to the second measurement being the same in both cases, whereas the accuracy of the measurements is relatively independent of the final state of the object. As in the case of measurements of the first kind, the requirement that the final state of the object reflect the inaccuracy of the measurement results would restrict the notion of quantum mechanical measurement in an unnecessary and undesirable way, by merging determinative and preparative aspects of measurement.

As said earlier a switch from an empiricist towards a more realist attitude regarding theory is motivated by the desire to get explanations of measurement phenomena in terms of objective properties of the object. In this section we have reached the conclusion that quantum mechanical properties (observables), either sharp or unsharp ones, are not to be considered as *objective* properties, and hence cannot contribute to such an explanation. This would imply that QM cannot furnish such explanations at all. Quantum mechanics seems to describe just the phenomena observed in measurements within the quantum domain. The reality behind the phenomena remains out of sight in the quantum mechanical description.

6. HIDDEN VARIABLES

Sometimes an empiricist interpretation of QM is equated with a denial of the existence of any reality that is not directly observed. Thus, it would be inappropriate to consider the microscopic objects themselves. Only the preparing and measuring instruments could be considered as real. For this reason hidden variables theories, purporting to

[14]Of course we do not want to diminish the importance of t he considerable insight into the structure of quantum mechanical propositions gained by the quantum logic approach.

describe the reality behind the directly observed phenomena, are often considered as metaphysical.

In our opinion such an estimation of the empiricist position is based on a confusion of the ontic and epistemic levels. Empiricism is an epistemic category, yielding information on one's attitude towards the significance of *theories*. This is a completely different issue from one's beliefs about reality itself. Thus, we may use thermodynamics as an instrument for calculating temperatures and pressures without being obliged to reject the existence of atoms. The confusion that is meant here is possibly caused by the idea that quantum mechanics is complete in the sense of being able to describe everything that exists. However, nothing compels the empiricist to surmise that quantum mechanics is complete in this sense. On the contrary, if he believes in the existence of electrons (as most physicists do) and other elementary particles that are never directly observed, then his empiricism may be an incentive for him to develop other theories than QM, to be used as instruments for describing features of reality that are not described by quantum mechanics. For this reason it is very well possible to entertain an empiricist interpretation with respect to QM, without considering hidden variables (HV) theories[15] as purely metaphysical. "Explanation" is a legitimate motive, next to "description of the phenomena", for development of theories. If QM does not provide sufficient explanation, then it is appropriate to device theories that are more promising in this respect. A belief in the ontic reality of a quantum world behind the appearances of the quantum mechanical measurement phenomena may be instrumental in the endeavour to describe this quantum world itself rather than restricting to the world of macroscopic preparation and measuring instruments. Hidden variables theories will for this reason in general refer not only to properties of preparation and measuring instruments (as is the case in an empiricist understanding of quantum mechanics), but *also* to properties of the object system. In order to be able to perform their explanatory task it is important that these latter properties can be considered as *objective* properties, being determined by the preparation of the object independently of what measurement is going to be performed. Such properties we will indicate with the generic name of "hidden variables". We next will have to clarify the relation between these hidden variables and the quantum mechanical observables that must be explained.

Let us introduce a hidden variables space Λ that can serve as a kind of phase space[16] for the object. A value λ of the HV corresponds with a point in this "phase space" Λ. It is generally supposed that it must be possible to prepare the object in a HV state characterized by a well–defined value of λ, and that a quantum mechanical state can be obtained by repeating this preparation procedure in such a way that $\rho(\lambda)$ is the relative frequency of the values of λ. It is further supposed that, if the object is prepared in a HV state determined by λ and subsequently is brought into interaction with a quantum mechanical measuring instrument, then this interaction must yield

[15]By "hidden variables theories" we not only mean possible completions of quantum mechanics by means of unobservable (hidden) variables, but we also include in this category theories like stochastic electrodynamics that are basically different from quantum mechanics but purport to reproduce the quantum mechanical results.

[16]We do not specify the dimension of Λ, which may be infinite in order to allow for the possibility that the hidden variable does not describe a particle but a field.

some (eigen)value of the measured observable. In the present section we will comply with this supposition, although it is far from clear whether such an assumption is warranted within the domain of quantum mechanics (see sect. 7.4.). The measuring process may be either a deterministic or a stochastic one. In the following we will discuss both versions.

6.1. Deterministic HV theories

In devising a HV theory that is able to reproduce the predictions of quantum mechanics (which seem to correspond very well with experimentally obtained results) we must be careful about the non–Boolean character of the latter theory. This makes it impossible to draw the analogy with classical statistical mechanics too far. Thus, it would not make sense to suppose the possibility of partitioning Λ for every quantum mechanical observable $\{E_k\}$ into regions $\Lambda(E_k)$ such that $\lambda \in \Lambda(E_k)$ would imply that measurement result k would be obtained with certainty if the measurement of observable $\{E_k\}$ would be performed. In such a deterministic theory the relative frequencies p_k of the quantum mechanical measurement results would be given by

$$p_k = \int_{\Lambda(E_k)} \rho(\lambda) d\lambda, \tag{58}$$

representing a completely classical statistical theory. Indeed, since in this theory to every value of λ a well–defined value is associated with *every* quantum mechanical observable the theory attributes to an object system simultaneously values to *incompatible* observables, thus frustrating the non–Boolean structure of QM.

We might hope that contextualism offers a way out of this problem.[52] Thus, if QM is describing only reality as it is in the context of a well–defined measurement setup, and the measuring instrument is able to influence the object, then the probability distribution $\rho(\lambda)$ may depend on the measurement context. Hence, (58) must be replaced by

$$p_k = \int_{\Lambda(E_k)} \rho_{\{E_k\}}(\lambda) d\lambda, \tag{59}$$

in which the suffix $\{E_k\}$ in $\rho_{\{E_k\}}(\lambda)$ indicates the dependence of $\rho(\lambda)$ on the measurement context of the $\{E_k\}$ measurement. Hence, for measurements of incompatible observables $\rho(\lambda)$ will be different in general, thus making it impossible to directly compare measurement probabilities of incompatible observables. In order to do so it would be necessary to calculate the influence the measurement has on $\rho(\lambda)$. In deterministic HV theories the hidden variables satisfy deterministic evolution equations. Thus, let us denote by $T_{\{E_k\}}(\lambda)$ the final value of the HV obtained during the measurement process of $\{E_k\}$ if the preparation yielded the initial value λ. Then, supposing the evolution of λ to be a Hamilton flow, we have $\rho_{\{E_k\}}(\lambda) = \rho(T^{-1}_{\{E_k\}}(\lambda))$, $\rho(\lambda)$ being the relative frequency with which λ is prepared prior to the interaction with the measuring instrument. Then the relative frequencies can be given as

$$p_k = \int_{T^{-1}_{\{E_k\}}(\Lambda(E_k))} \rho(\lambda) d\lambda, \tag{60}$$

in which $T^{-1}_{\{E_k\}}(\Lambda(E_k))$ is the set of initial HV values that is mapped onto $\Lambda(E_k)$ by the measurement process. If the preparation is independent of the measurement that is following it, $\rho(\lambda)$ can now be taken the same in different experiments.

If by the mapping $T_{\{E_k\}}$ the "phase space" would be mapped *onto* Λ in a one–to–one manner, contextualism would not work as a means to evade the problem, because even now for every initially prepared value of λ and every observable $\{E_k\}$ an unique final value would be defined, attributing an unique value to each observable as in the non–contextualistic case. However, the contextualistic model leading to (60) is too restrictive in the sense discussed in sect. 5., viz., attributing the value of the observable as a property to the object system via the final value $\lambda_f = T_{\{E_k\}}(\lambda)$ of the hidden variable. In a more sophisticated model the observable could be attributed to the measuring instrument as is done in the empiricist interpretation of quantum mechanics. We should then attribute also a HV μ to the measuring instrument, and attribute the value of the observable to the final value μ_f of μ. Thus, if $S_{\{E_k\}}$ describes the mapping of the (deterministic) interaction between object and measuring instrument, then

$$(\lambda_f, \mu_f) = S_{\{E_k\}}(\lambda, \mu), \tag{61}$$

and

$$\rho_{\{E_k\}}(\lambda, \mu) = \rho(S^{-1}_{\{E_k\}}(\lambda, \mu)). \tag{62}$$

If Λ^A_k is the part of the apparatus "phase space" Λ^A yielding measurement result k, we obtain for the relative frequencies

$$p_k = \int_{\Lambda^A_k} d\mu \int_{\Lambda} d\lambda \rho(S^{-1}_{\{E_k\}}(\lambda, \mu)) = \int_{S^{-1}_{\{E_k\}}(\Lambda^A_k \times \Lambda)} d\mu d\lambda \rho(\lambda)\rho^A(\mu), \tag{63}$$

$\rho^A(\mu)$ being the initial distribution of μ. The impossibility of an objective attribution of a value of the observable $\{E_k\}$ to the object through its initial HV λ is now warranted by the fact that the same initial value of λ may lead to different values of k if combined with different initial values of μ.

Although the possibility of such a deterministic contextualistic solution cannot be denied, it is not a very attractive one. As a matter of fact, this solution would leave unexplained why certain preparations, described (in case of a PVM) by eigenfunctions of the measured observable, yield with certainty one well–defined (eigen)value of the observable, although the initial variable μ may vary from one realization of the measurement to another arbitrarily over the whole range allowed by the initial preparation of the measuring instrument. An explanation of measurement statistics in terms of different values of μ is physically not different from taking into account a nonideality of the measurement procedure as discussed in sect. 3.: fluctuations in the measurement outcome are, in case of sharp preparation, due to fluctuations in the measuring instrument, i.e., to measurement noise.[(33)] Hence, an explanation along these lines would be appropriate for *nonideal* measurements, but not for ideal measurements corresponding with PVM's.

6.2. Stochastic HV theories

In a HV theory in which it is thought possible to prepare the object in a HV state determined by λ, a measurement of some observable $\{M_k\}$, representing a *nonideal* measurement of some other observable $\{E_k\}$, could be characterized by a conditional probability $p_{\{M_k\}}(k \mid \lambda)$, describing the probability of finding measurement result k if the object was prepared with HV λ. For such measurement processes the relative frequency p_k of the measurement results can be given as

$$p_k = \int_\Lambda \rho(\lambda) p_{\{M_k\}}(k \mid \lambda) d\lambda. \tag{64}$$

In an *ideal* measurement of the observable $\{E_k\}$ it might be supposed that the conditional probability $p(k \mid \lambda)$ coincides with the indicator function $\chi_{\Lambda_k}(\lambda)$ of some region Λ_k of Λ to which the value k of $\{E_k\}$ can be attributed. Then a deviation of $p_{\{M_k\}}(k \mid \lambda)$ from such an indicator function can be explained by the measurement dynamics being subject to stochastic fluctuations sending the pointer into the "wrong" final state. Taking λ_{mn} as in equation (11), relation (13) between the probabilities of an ideal and a nonideal measurement can be obtained if the analogous relation holds for the conditional probabilities, viz.,

$$p_{\{M_k\}}(m \mid \lambda) = \sum_n \lambda_{mn} \chi_{\Lambda_n}(\lambda). \tag{65}$$

The expression (64) is generally used in so–called *stochastic* HV theories. It was demonstrated in Ref. 53 that, under certain conditions, (64) can be obtained for both HV theories in which k is referring to a property of the object and theories attributing the observable to the measuring instrument. It is of a contextualistic nature in as far as the conditional probability $p_{\{M_k\}}(k \mid \lambda)$ depends on the way the observable is measured. By Eberhard[54] the expression (64) is considered as the most general one to be obtained in a HV theory.

It was pointed out by Suppes and Zanotti[55] that the strict EPR correlations require a deterministic theory. Indeed, such strict correlations would not be found if the measurement process would induce so much stochasticity that different measurement results (pointer states) could result from the same initial value of the hidden variable. Since, for this reason, ideal measurements yield a better representation of the EPR correlations than the nonideal ones, we cannot restrict to nonideal measurements (as was done in sect. 4.), but should focus on the ideal measurements that are usually considered. Note that the Suppes–Zanotti requirement of determinism does not exclude a certain amount of stochasticity even in ideal measurements. There need not be a deterministic relation like (61) between the initial and final values of the hidden variables. It is sufficient that the measurement dynamics is such that there are no stochastic fluctuations causing the "wrong" measurement result to be obtained, i.e., that μ_f is in an incorrect Λ^A_k.

Yet, it seems that this requirement of determinism does pose a real problem to the attempt at obtaining a HV theory that is capable of reproducing the non–Boolean structure of QM. As a matter of fact, due to the Suppes–Zanotti requirement of determinism it seems necessary that it be possible to attribute well–defined values of incompatible observables to the object already as it is objectively prepared in a HV state determined by λ in the sense that such a value is obtained with certainty if the measurement of the corresponding observable is carried out. This would seem to imply that the more sophisticated HV theories do not perform any better than the simple deterministic one, or even than an objectivistic realist interpretation of QM: in all cases the set of observables acquires a Boolean rather than a non–Boolean structure. In sect. 7. we shall return to this problem and discuss a condition enabling a HV theory to avoid such a Boolean structure.

6.3. The Bell inequalities in HV theories

As is well known the principal reason for Bell to derive his inequalities(36) was the desire of being able to discriminate between quantum mechanics and so–called hidden variables theories purporting to provide a (more) complete description of reality. However, Bell was convinced that Bohm(56) had succeeded in developing a hidden variables theory that is equivalent with QM in all observational respects, be it that this HV theory was of a *nonlocal* nature. For this reason Bell restricted his attention to *local* HV theories, and for such a theory he derived his inequalities. He always maintained that the locality assumption is crucial in deriving a result that violates QM. As a consequence of this it is now quite generally accepted that the reality underlying QM is of a nonlocal nature.

As discussed in sect. 4. we do not think that locality is *essentially* involved in a derivation of the Bell inequalities. For this reason we want to consider critically in the present section the role of the locality assumption (LOC). Strictly speaking, from a violation of the BI it is only possible to infer nonlocality if LOC would be the *only* assumption in the derivation of the inequalities. If there would be other assumptions apart from LOC, it would be possible to blame the violation of the BI on one of these other assumptions rather than on the violation of LOC. Let us first stipulate that the assumption of "determinism" (DET), as considered in sect. 6.1., cannot serve such a purpose, because derivations of the BI can be given as easily for stochastic HV theories as for deterministic ones. In the following we therefore shall deal in an explicit way with the general stochastic theory (i.e. nonideal measurements), the deterministic case (i.e. ideal measurements) being just a special case.

The existence of a quadrivariate jpd p_{ijkl} (cf. sect. 4.) is sufficient for a derivation of BI. Within the context of a HV theory in which it is supposed possible to prepare the object in a HV state with a well–defined value of λ, and perform subsequently a joint measurement of the four observables, this could be translated into the assumption of the existence of a conditional jpd $p(ijkl \mid \lambda)$, satisfying

$$p_{ijkl} = \int_\Lambda \rho(\lambda) p(ijkl \mid \lambda) d\lambda. \tag{66}$$

In this and the following expressions the dependence of the probabilities on the measurement arrangement is suppressed in the notation. If applied to the measurement arrangement of Fig. 2 no locality property would be required of the conditional jpd. In case of the EPR–like measurement arrangement of Fig. 3, it would be appropriate to require the locality condition

$$p(ijkl \mid \lambda) = p(ij \mid \lambda)p(kl \mid \lambda), \tag{67}$$

describing the conditional independence, given a value of λ, of the two detection processes (compare (55)). It is clear that it is not the locality property (67) that is responsible for the possibility of deriving the BI. These would hold also if there would be nonlocal interactions between the measurements in the left and right arms of the interferometer, as long as (66) is valid.

The validity of (66) hinges on the joint nonideal measurement of the four observables that are involved. It was discussed in sect. 4. in what sense the usual EPR measurements, involving the joint (ideal) measurement of two PVM's, can be considered as joint nonideal measurements of the four observables. It was also found there that in this sense the usual four EPR measurements define four *different* quadrivariate jpds. This can now be explained by the dependence of the conditional probabilities $p(ij \mid \lambda)$ and $p(kl \mid \lambda)$ on the measurement arrangement that is actually present.

As was argued in sect. 4.2., within the purely empiricist interpretation of QM this dependency prevents a derivation of the BI. However, within the context of HV theories we have more possibilities in this respect. Thus, according to the HV theory yielding (66) and (67) the bivariate probability distributions p_{ik}, etc. of the usual EPR experiments can be written as

$$p_{ik} = \int_\Lambda \rho(\lambda)p(i \mid \lambda)p(k \mid \lambda)d\lambda, \tag{68}$$

in which the conditional probabilities $p(i \mid \lambda)$ etc. refer to the ideal measurements of the PVM's. Now, in order that the probabilities (68) satisfy the BI (33) it is not necessary that the quadrivariate jpd, warranting this, is actually realized in the physical process. It would be sufficient for a derivation of the BI if some quadrivariate jpd exists from which the bivariate jpd's can be derived *mathematically.* In the theory presented here such a jpd might exist,[(53)] viz.,

$$p_{ijkl} = \int_\Lambda \rho(\lambda)p(i \mid \lambda)p(j \mid \lambda)p(k \mid \lambda)p(l \mid \lambda)d\lambda, \tag{69}$$

in which the conditional probabilities are the ones from (68) and its analogues. All bivariate jpd's (68) can be obtained from (69) by means of partial marginalization, because the conditional probabilities satisfy $\sum_i p(i \mid \lambda) = 1$, etc..

The derivation of the BI presented here is a very general one, and seems to highlight in a dramatic way the role played by the locality assumption in its derivation. Indeed, if locality does not hold the univariate conditional probabilities figuring in (68) would depend on which observable is measured jointly with it in the EPR experiment, thus preventing the existence of a single quadrivariate jpd (69) for all four EPR measurements. Therefore it seems impossible to evade the conclusion that local HV theories of the kind discussed above inevitably imply the BI, and, hence, are in conflict with quantum mechanics. If the considered class of theories were the most general one, then this indeed would warrant the conclusion that the reality underlying quantum mechanics is a nonlocal one.

However, in the following we want to discuss some ideas that, if adequate, would put the generality of the foregoing analysis into doubt. Since these ideas are concerning *additional* assumptions, apart from locality, that are made in the present section, this would open the possibility of blaming the BI on these extra assumptions rather than on locality.

As regards additional assumptions contributing to the existence of the quadrivariate jpd (69) two possibilities may be mentioned here. The first one is of a mathematical character, and is concerned with the mathematical existence of the integral (69). The conditional probabilities $p(i \mid \lambda)$, etc., need not be smooth functions of λ. In fact they may be distributions, not allowing a multiplication as performed in order to obtain (69). A discussion of possible physical explanations of this must be postponed to future work. In the remainder of this article we want to stress a second possible origin of the failure of a quadrivariate jpd to exist for EPR experiments. Our basic consideration stems from the observation that, in order to define the quadrivariate jpd (69) it is necessary to combine into one single quantity measurement results obtained in *different* experiments. Stated differently, in order that (69) makes sense it is necessary to assume that the *same* value of λ can be prepared as an initial HV value in each of the four EPR experiments. This was already discussed by De Baere,[57, 58] and indicated as a *reproducibility hypothesis* (RH). In sect. 7. RH will be discussed more extensively. Here it may suffice to remark that in the deterministic case in which the measurement result is determined uniquely by the initial value of λ, RH implies that it must be possible to attribute to the object a well–defined value of all four observables that are involved in the BI, even in a state preceding any measurement. In the case of nonideal measurements there is a certain relaxation of this implication because of the stochasticity of the measurement. Yet, also here RH is presupposed if we attribute to the object in its initial state the four conditional probabilities $p(i \mid \lambda)$, etc., of the four observables[17]. If RH would not be an allowed hypothesis within the domain of quantum mechanics, this would mean that a derivation of the BI along the lines of the present section would not have any physical relevance for quantum mechanical measurements.

7. HIDDEN VARIABLES AND COUNTERFACTUALS IN PHYSICS

7.1. Some considerations on the program of completing quantum mechanics by hidden variables

Why hidden variables?

QM gives, in general, only predictions for the statistics of measurement outcomes and is unable to predict the outcomes of single measurement acts. It may be argued

[17]This was indicated as a "quasiobjectivistic" realism in Ref. 53, in order to indicate that it is not the observables that are attributed to the object but only the conditional probabilities.

legitimately that, as long as a theory has this (statistical) characteristic, it is not a complete theory. In a more complete theory it should be possible, for instance, to give a detailed account also of individual processes and events. Because the statistics obeys quantitative laws, it seems an obvious extrapolation to postulate also a lawful behavior, and the existence of a corresponding formalism, at the individual level (in much the same way as classical mechanics is the substructure for classical statistical mechanics).

As it is a natural requirement of a completion of some theory (e.g. QM) that it provides more specific predictions, it should be based necessarily on more detailed initial conditions. This unavoidably entails the introduction in the new formalism of *more* variables than are used in the older one. This program usually goes under the name of the HVs program, the HVs constituting the extended set of variables in terms of which a more complete state may be defined. The essentials of such a formalism should be constituted by a HV state, which we shall denote by $|\lambda)$, and a corresponding (set of) dynamical equation(s). The state $|\lambda)$ will be a better (i.e. more faithful) representation of processes which may be thought to occur at the ontological level. Possibly, these supplementary variables will correspond, in the future, to a greater control over the initial conditions during the preparation procedure, or to degrees of freedom which are assumed to be useful at a deeper level of description.

However, from an empirical point of view there is, at present, no need at all for a HV formalism. The control over individual quantum preparations is not (yet ?) sophisticated enough as to allow for deterministic individual predictions in general (including also incompatible observables). Furthermore, also no simple statistical facts are known which contradict QM, and which could give useful hints as to the kind of new principles which should underlie a new formal scheme. The necessity of such a HV formalism is mainly theoretical, as already noticed in sect. 5.1. with respect to the existence of certain experimental correlations, which are perfectly well described by quantum mechanics, but which seem to need such a HV formalism in order to provide an explanation. In particular the EPR correlations have induced a vast literature, related to the BI, which is based on considerations on individual quantum events. In order to be physically reliable and meaningful, such considerations should be based preferably on a corresponding quantitative HV formalism.

Hidden variables: return to classical physics?

With respect to the execution of the program of completing QM by means of a HV formalism it is worthwhile to remember that already long ago W. Heisenberg was of the opinion that, if QM were to be replaced some day by another, more successful, theory, this theory should probably differ from classical mechanics still more than QM differs from it. Now, when looking at the current literature on HV completions of QM, it is striking that these have almost always a purely classical basis, contrary to Heisenberg's suggestion. For instance, Kochen and Specker (KS)[59] start their well known paper with the following definition: "...the problem of hidden variables ...is the possibility of imbedding quantum mechanics into a classical theory ...[which] ...remains a controversial and obscure subject".

A complete return to classical ideas underlying HV schemes was also the basis of Bell's original work, and of all subsequent similar ones. However, because the quantum formalism has precisely been introduced as a substitute for a deficient classical theory, the failure of the HV program, as defined above, could be foreseen. The common characteristic of the ensuing argumentations (which are either of the EPR–Bell or of the KS type) is the incompatibility of their assumptions with QM: either a Bell–type inequality is violated by empirical facts which verify QM, or, as in the KS case, the results arrived at are contradictory. This means that in the search for an analogy between QM and CM at least one basic classical property should be given up.

As mentioned before, the present opinion on that point is that the locality principle is no longer valid in the domain of individual quantum processes. However, another important (though not fully recognized up to now) characteristic taken over from CM, in addition to the locality assumption, is the assumption of *reproducibility* of individual (HV) states $|\lambda)$ in localized regions of space–time, i.e. of individual physical states in different experiments (cf. sect. 6.3.). In the following it will be shown that in the classical domain this assumption allows for the justified use of counterfactual ideas which usually also underlie the derivations of the BI in the quantum domain. It follows that the validity of locality in the domain of individual quantum events may be retained, provided one gives up the validity of reproducibility of individual subquantum states $|\lambda)$ or, stated otherwise, the unrestricted use of counterfactual argumentations.

7.2. General conditions for the justified use of counterfactual reasoning in physics

General considerations

Counterfactual reasoning deals with nonactual physical processes and events and plays an important role in physical argumentations. In such reasonings it is assumed that, if some set of manipulations were carried out, then the resulting physical processes would give rise to effects which are determined by the formal laws of the theory applying in the envisaged domain of experimentation.

The physical justification of counterfactual reasoning depends on the context in which it is used. Rigorously speaking, given some theoretical framework, such reasoning is *always* allowed and justified as soon as one is sure of the *possibility* of *at least one* realization of the pre–assumed set of manipulations. In general, in counterfactual reasoning it is even understood that the physical situations to which the reasoning applies can be reproduced at will, and hence may be realized *more than once*.

It appears that the justification of counterfactual reasoning requires two essential conditions to be satisfied:
1) the availability or existence of some theoretical scheme, and
2) the *actual* reproducibility (i.e. not only the *theoretical* possibility) of the initial physical conditions corresponding to some prepared state.
It may be seen that e.g. our reliance on all our technical achievements (as well in the classical as in the quantum domain) is based on the justified use of such counterfactual reasoning, i.e. because of the fulfillment of the conditions above which will be discussed in detail below.

Condition TH: Availability of a theory

Within the context of an existing theory (e.g. classical or quantum), counterfactual statements concern the *causal* and *deterministic* relationship between sequences of events which are allowed by the theory. Counterfactual statements, hence, concern always the theoretical prediction of outcomes of measurements for a given set of initial conditions, corresponding to some possible preparation procedure or to a selection after a measurement process. It is clear that such statements are only possible on the basis of the availability of a theory describing these causal and deterministic relationships.

If the theory is for single systems (e.g. CM), then the relationship concerns two events. If, on the contrary, the theory is for ensembles and of the statistical type (e.g. QM), then, for any preparation which has a representation in the theory (e.g. by means of a state vector), the relationship concerns frequencies of outcomes, or quantities (such as probabilities) which are defined in terms of these frequencies.

In the above definition of counterfactual statements, causality refers to the cause–effect character of the relationship between the (sequences of) events, and the requirement of determinism is essential because only then the relationship may be verified empirically. Hence, the properties of causality and determinism are essential preconditions for discussing the physical relevance and justification of a counterfactual statement.

Condition RH: Hypothesis of reproducibility of initial conditions

In order that counterfactual reasoning is physically justified, also the condition of reproducibility must be fulfilled. It is important to realize that the *possibility* of reproducing a set of initial conditions does not follow from the theory itself: it is only our experience with the real world which actually assures us of the possibility of *more than one* actual realization of an initial condition. Because our main *physical* experience with (and interest in) the real world intuitively concerns (at least up to now) reproducible phenomena (at the observational level), the validity of this reproducibility is always *implicitly* assumed when making counterfactual reasonings. Yet, when looking at processes in the sub–quantum domain of *individual* quantum phenomena, this self–evident looking reproducibility property should be approached with some care, i.e. one should not exclude a priori the possibility that in these not completely controllable domains of experience RH is no longer valid. One reason for this is on account of sound scientific scepticism, while another one is more physical and derives from our experience that the world as a whole is actually *not* reproducible either for practical reasons (like e.g. the infinite dimensionality of state space), or because of certain, as yet unexplained, restrictions on the possibility of reproduction of initial states within certain domains of experimentation (e.g. the quantum domain). Although, up to now, this nonreproducibility has played no role at all in our theoretical account of the world, it cannot be excluded that the (im)possibility of reproduction of initial states may have consequences in certain domains of experimentation. It is this (im)possibility which will be investigated furtheron in the description of quantum phenomena at the level of individual quantum events.

In physical terms, RH concerns the reproducibility, in a bounded region of space–time, of the physical states of some theory. For our existing theories this means quantitatively that identical preparations should be represented by identical initial states, resulting in identical outcomes if the same observable is measured. In these theories the (observational) meaning of "identical" (preparation and outcome) is determined by the theory itself, e.g. in CM identical states imply identical *single* outcomes, whereas in QM identical states allow only for identical *frequencies* of the outcomes[18]. In CM and QM reproducibility in these two different senses is taken for granted. However, for any other future theory, such as a HVT for the individual quantum case, the reproducibility of the individual initial state $|\lambda)$, i.e. the possibility of identical states $|\lambda)$ in different measurements, must have initially the status of a *hypothesis*, and it remains to be investigated whether this hypothesis is compatible with observed facts. Although it is the validity of RH in consecutive individual quantum events that will be investigated, the observed facts which will be of interest in the present work are not the outcomes of individual quantum events but the frequencies of outcomes within ensembles of such events. It will be investigated whether these frequencies are compatible with a HVT in which RH is valid.

It is also important to be aware of the fact that identity or reproducibility of the states of some theory (such as CM and QM) does not necessarily correspond to ontological identity. In view of the situation in QM (namely, that identical QM states $|\psi\rangle$ do not, in general, result in identical outcomes) it is rather the contrary situation that should be expected, i.e. identical quantum states do not correspond to identical individual ontological situations (represented in a concise way by "ontological" states $|\omega)$) for the members of the ensemble. Also, it is to be hoped for that at each deeper level of description the relevant states $|\lambda)$ are more faithful representations of the real, ontological, states $|\omega)$. Below it will be shown that for the HV states $|\lambda)$ RH is not valid. This immediately implies that neither the ontological states $|\omega)$ are reproducible.

On the requirement of actualizability of theoretical possibilities

The idea developed above about the connection between RH and the justified use of CFD is inspired by the basic requirement that the general purpose of any theoretical scheme is to make unambiguous (i.e. deterministic) predictions about future *actual* measurements, given *actual* past or present initial conditions (represented by the theory's symbols, such as a state vector). The emphasis on actualizability of physical situations and processes is essential because it is only under these circumstances that the theory may be verified. Hence, within some quantitative framework, the only relevant and justified way to formulate counterfactual statements is to reason with in principle possible *actualizable* situations.

However, in almost all derivations of the BI the counterfactual argumentations concern situations which are in principle *nonactualizable*, such as reconsidering the outcome of a past actual measurement process in an EPR experiment under alternative conditions in a space–like separated region. According to the locality condition the outcome in the hypothetical EPR experiment should remain unchanged under the alternative conditions. It is immediately seen, however, that without imposing any further conditions on the possible future actualizability of these hypothetical events, such reasonings are completely irrelevant.

As was indicated in sect. 6.3., and as will be discussed more extensively in sect. 7.4.,

[18]In an empiricist interpretation of quantum mechanics it might be preferable referring to "identical preparations" rather than to "identical (initial) states", described by some wave function.

it is the validity of RH which has to be imposed in these derivations of the Bell inequalities: it is under this extra condition that it may be assumed that it makes sense to identify in two different EPR experiments like $\{E^1_{k_1}E^2_{k_2}\}$ and $\{E^1_{k_1}F^2_{l_2}\}$ (cf. sect. 4.2.) identical individual measurement results for $\{E^1_{k_1}\}$. If the individual initial conditions (preferably to be formulated quantitatively in terms of HVs) of the $\{E^1_{k_1}\}$ measurement would not be realized (i.e. actualized) as exactly identical initial conditions in the two measurements, counterfactual argumentation would be physically irrelevant because then the values of $\{E^2_{k_2}\}$ and $\{F^2_{l_2}\}$ need not have any relation to each other.

It is clear that, unless RH is invoked to guarantee the repeated actualization of the individual initial state, such situations concern alternative worlds and are completely inaccessible to experimental verification. Counterfactual reasoning often deals with hypothetical events rather than actual ones. Thus, in the context of the $\{E^1_{k_1}E^2_{k_2}\}$ measurement the hypothetical event is sometimes considered that, given an initial individual state and a measurement result of $\{E^1_{k_1}\}$ a certain value of $\{F^2_{l_2}\}$ is obtained rather than of $\{E^2_{k_2}\}$. As demonstrated in sect. 4.2., within the domain of quantum mechanics the disturbing influence of the $\{E^2_{k_2}\}$ measurement setup will yield in general a different value for the hypothetical $\{F^2_{l_2}\}$ event than if the latter observable is actually measured itself. This example stresses the importance of the requirement that only *actual* events must be considered: hypothetical events may be governed by different physical laws from the ones actual events obey.

Nonreproducibility vs. nonlocality

Violation of the Bell inequalities is often attributed to an influence by action–at–a–distance on outcomes in correlated measurements. However, it should be kept in mind that the purpose of physical theories is to formulate causal relationships between events called a "genuine" preparation, from which physical influences propagate to some measurement apparatus, and the measurement proper. Therefore, we find it not opportune to consider, in correlated *measurement* events, one such event as a kind of preparation for the occurrence of the other, correlated, event and to search for eventual causal relationships between both events. This amounts to saying that because wrong questions are asked, wrong answers are to be expected. Moreover, it is our opinion that in proceeding in this wrong way the confusion is still more emphasized in adhering to the orthodox idea that the selection of a measurement result is completely stochastic, in the sense of originating "out of the blue" without any reason whatsoever. It is clear that the combination of asking the wrong question and adhering to some questionable (be it orthodox) view leads immediately to the necessity of the existence of influences by action–at–a–distance. This "orthodox" analysis of the significance of correlated events will be criticized in more detail in the next sections.

In our further work we will, therefore, try to ask the right questions and investigate whether locality may be retained in a universal way in correlated quantum processes and try to resolve the Bell issue in an alternative way. We will start from the idea that measurement results are not generated spontaneously, but may be associated with a preparation of the object in a HV–state $|\lambda)$. It then follows that the former, erroneously formulated, causal independence of a measurement result in one region from the correlated measurement process in a space–like separated region, may be reformulated by the condition that if the underlying HV state $|\lambda)$ is reproduced, this gives rise to an identical measurement outcome in the localized region in which the measurement arrangement is left unchanged, even if the experimental arrangement has been changed at a space–like distance. Taking this reformulation as a starting point allows for an analysis of the Bell issue from quite a different perspective. It may very well be that not the alleged nonlocality is responsible for the violation of the BI by quantum mechanics, but that this violation must be accounted for by the lack of repeatability of the physical conditions on a HV–level (i.e., their "nonreproducibility") in different quantum mechanical experiments. However, because such a repeatability ("reproducibility") condition can in no way be equated with a locality condition, locality may retain its universal status, in full agreement with all present observations.

Counterfactual definiteness

Closely connected with this counterfactual reasoning is the stronger idea of counterfactual definiteness in the sense this is used in relation to the Bell inequalities. According to CFD the validity is assumed of argumentations in which *actual* (in fact: *possible* but yet actualizable) events together with *hypothetical* ones are combined, even if the hypothetical events may *never* be actualized.

A typical such hypothetical case concerns e.g. an actual individual quantum event for which it is tried to answer the question "what would have been the outcome of the measurement if in some space–like separated region an experimenter had made another choice for certain adjustable parameters". It is clear that, unless further constraints are imposed on the theory, such a question may never be answered because an actual event which happens at a certain time (as a result of a certain ontological physical process) can happen only once. However, as mentioned already above, the circumstance that the actual and the hypothetical events are both allowed by the theory is not sufficient to guarantee the physical relevance of argumentations in which the hypothetical event is used. This demand of physical relevance requires that the hypothetical event is also a possible actual event in the future. The assumption that this is actually possible constitutes the essence of the condition RH.

Again, it is seen that the physical relevance of hypothetical events, which are furthermore impossible to carry out, depends on the availability of a theoretical scheme which allows for the reproducibility of the initial states $|\lambda)$ which represent "identical" individual preparations. As with CM and QM it is the domain of application of the new theory, and not the theory itself, which determines whether RH is applicable or not. If it should turn out that RH is not valid, then this should be built in in the new theory.

It is the general validity of such stronger use of counterfactual reasoning which will be investigated in detail below. To this end the conflict between the BI and QM will be re–investigated from the point of view of the justified use of CFD. It will be shown that the assumption of the breakdown of this justified use of CFD in the sub–quantum (or: single quantum system) domain is a natural explanation for the above–mentioned conflict. In terms of a (hypothetical) sub–quantum formalism for the description of nature's behavior this amounts to the breakdown of the property of individual reproducibility of initial conditions. This might mean that not only the world as a whole but also *localized* physical states are nonreproducible, and this might imply also the nonreproducibility of ontological reality, i.e. reality as it exists "out there" independent of an observer's observations. A less far–reaching conclusion would be that nonreproducibility of the ontological HV state must hold within the domain of application of quantum mechanics, e.g., in the sense of the impossibility of reproducing the same HV state in the different contexts of the measurements of incompatible quantum observables like the EPR observables $\{E^1_{k_1}E^2_{k_2}\}$ and $\{E^1_{k_1}F^2_{l_2}\}$, leaving open, for the moment, the question of the possibility of RH outside this domain.

7.3. The use of CFD in classical physics

Discussion of the physical relevance of the BI for correlated classical states

As will be discussed in the next section, it are argumentations of the counterfactual type on *individual* quantum events which give rise to the issues (e.g. quantum nonlocality) connected with the BI in a quantum context, or with the EPR reasoning against the completeness of QM. However, in the quantum domain generally ensembles are needed in order to compare quantum predictions with empirical data. This is reflected in the BI where the relevant quantity is a correlation function.

with the purpose to get further insight into the deeper origin of these issues we will consider in this section the case of an ensemble of correlated individual classical systems S_1 and S_2 with angular momenta $\bar{J}_{1,i}$ and $\bar{J}_{2,i} = -\bar{J}_{1,i}$ $(i = 1, \ldots, N)$, respectively. The correlation may originate e.g. from the decay of a system S, prepared in the spherically symmetric classical angular momentum state $\left|\bar{J} = 0\right)$, in systems S_1 and S_2. A future measurement of the component of the angular momentum of e.g. S_1 along *any* direction $\bar{a}$ is then well determined and predicted to have the value $m_1(\bar{a}) = J_1 \cos(\bar{1}_{J_1}, \bar{a})$. From this expression it is seen that this value depends only on localized quantities and not on any process that happens in spacelike separated regions. Hence, the selection of the measured value may be assumed to be the result of localized physical processes. Also, the physical relevance of the prediction for *any* direction, for a given classical state $\left|\bar{J}_1\right)$, rests on the property of reproducibility of the state $\left|\bar{J}_1\right)$, i.e. on the circumstance that different physical situations may be represented by the same state $\left|\bar{J}_1\right)$.

Assume, then, that on each member i of the ensemble the angular momentum component $m_{1i}(\bar{a})$ along $\bar{a}$ of system S_1 is measured in a region R_1, and in a space–like separated region R_2 the angular momentum component $m_{2i}(\bar{b})$ along $\bar{b}$ of system S_2. Define the following correlation function[60]:

$$C(\bar{a}, \bar{b}) = \frac{1}{N}\sum_{i=1}^{N} m^{(1)}_{1i}(\bar{a}) m^{(1)}_{2i}(\bar{b}), \tag{70}$$

where the superscript "(1)" refers to the couple $(\bar{a}, \bar{b})$ of analyzer directions, and, assuming $\left|\bar{J}_1\right| = 1$, we have that

$$m^{(1)}_{1i}(\bar{a}) = \cos(\bar{1}_{J_1,i}, \bar{a}) \tag{71}$$

and

$$m_{2i}^{(1)}(\bar{b}) = \cos(-\bar{1}_{J_1,i}, \bar{b}). \tag{72}$$

A typical counterfactual question is of the form: "what would have been the result if, instead of measuring along $\bar{a}$, one had measured along $\bar{a}\,'$" while retaining the same conditions in the region R_2 (i.e. the same parameter $\bar{b}$ and the same state $\left|\bar{J}_{2,i}\right) = -\left|\bar{J}_{1,i}\right)$ for S_2, both characterizing the local situation). Assuming that the classical theory applies to the hypothetical case (i.e. case no. 2, referred to as superscript "(2)" in eqs.(73) and (74) below), one may write down the relations

$$m_{1i}^{(2)}(\bar{a}') = \cos(\bar{1}_{J_1,i}, \bar{a}'), \tag{73}$$

$$m_{2i}^{(2)}(\bar{b}) = \cos(-\bar{1}_{J_1,i}, \bar{b}). \tag{74}$$

The crucial point is that the hypothetical situation can actually be realized, i.e. CM allows for *a new realization* (i.e. a "copy") of an already *realized* state, both of the object state $\left|\bar{J}_{2,i} = -\bar{J}_{1,i}\right)$ and of the apparatus state which in this case is characterized by the parameter $\bar{b}$. It appears that, if the universal validity of LOC is assumed from the outset, the key issue becomes *reproducibility* of the state representing individual physical situations in a localized region of space–time (characterized in CM by the relevant vectors $\bar{1}_{J_2,i}$ and $\bar{b}$). Obviously this requirement is satisfied by CM and justifies counterfactual reasoning using individual situations, i.e. the combination of actual and of hypothetical measurement outcomes is physically relevant. On these grounds one may equate the rhs. of eq. (74) with $m_{2i}^{(1)}(\bar{b})$, so that instead of (74) one may write

$$m_{2i}^{(2)}(\bar{b}) = \cos(-\bar{1}_{J_1,i}, \bar{b}) = m_{2i}^{(1)}(\bar{b}). \tag{75}$$

For the i–th single actual couple of systems S_1, S_2 two further hypothetical situations, distinguished by superscripts "(3)" and "(4)", may then be considered: in case "(3)" measurements along the couple of directions $(\bar{a}, \bar{b}')$ are counterfactually considered, and in case "(4)" this is done for the couple $(\bar{a}', \bar{b}')$. If for these cases the above argumentation (as for case "(2)") is repeated then one is lead to the relations:

$$m_{1i}^{(3)}(\bar{a}) = \cos(\bar{1}_{J_1,i}, \bar{a}) = m_{1i}^{(1)}(\bar{a}), \tag{76}$$

$$m_{2i}^{(3)}(\bar{b}') = \cos(-\bar{1}_{J_1,i}, \bar{b}'), \tag{77}$$

$$m_{1i}^{(4)}(\bar{a}') = \cos(\bar{1}_{J_1,i}, \bar{a}') = m_{1i}^{(2)}(\bar{a}'), \tag{78}$$

$$m_{2i}^{(4)}(\bar{b}') = \cos(-\bar{1}_{J_1,i}, \bar{b}') = m_{2i}^{(3)}(\bar{b}'). \tag{79}$$

Denote by $E^{(r)}, r = 1, \ldots, 4$ the ensemble in which one of the measurement arrangements is actually set up. From the relations (75), (76), (78), and (79) the BI may be derived by using the Stapp–Eberhard–Peres[43, 23] procedure: for each single actual case labeled by the index i (e.g. in the ensemble $E^{(1)}$), and for the three corresponding hypothetical cases (distinguished by superscripts "(2)", "(3)", and "(4)") the following expression, whose value has an upper bound 2, is considered:

$$m_{1i}^{(1)}(\bar{a})m_{2i}^{(1)}(\bar{b}) + m_{1i}^{(2)}(\bar{a}')m_{2i}^{(1)}(\bar{b}) + m_{1i}^{(1)}(\bar{a})m_{2i}^{(3)}(\bar{b}') - m_{1i}^{(2)}(\bar{a}')m_{2i}^{(3)}(\bar{b}') \leq 2. \tag{80}$$

Summing over all the individuals of the ensemble leads to the BI for the correlation functions (70):

$$C(\bar{a},\bar{b}) + C(\bar{a}',\bar{b}) + C(\bar{a},\bar{b}') - C(\bar{a}',\bar{b}') \leq 2. \tag{81}$$

In this inequality the correlation functions may be considered as *actual* because its derivation was based on the use of valid counterfactuals, i.e. because the formalism of CM satisfies a *reproducibility* condition of *single* physical states.

Alternative argumentation

In order to clarify further the justified use of CFD or its physical counterpart RH one may set up the following alternative but equivalent reasoning. Consider again four ensembles $E^{(r)}$ $(r = 1, \ldots, 4)$ each consisting now of N *actual* identical classical systems S (instead of one actual and three counterfactual ensembles as above). Assume again that each system S is prepared in the classical singlet state $\Big|\bar{J} = 0\Big)$, and that it disintegrates in subsystems S_1 and S_2.

Assume, then, that on the first ensemble, $E^{(1)}$, the components along directions $\bar{a}$ and $\bar{b}$ are measured with results resp. $m_{1i}(\bar{a})$ and $m_{2i}(\bar{b})$, $i = 1, \ldots, N$. The components along any other directions for both systems, e.g. $m_{1i}(\bar{a'})$ and $m_{2i}(\bar{b'})$, $i = 1, \ldots, N$ may then be considered counterfactually. The counterfactual assignment is justified for the same reasons as explained in setting up the BI (81).

On the second ensemble, $E^{(2)}$, one may consider e.g. actual values $m_{1i}(\bar{a})$ and $m_{2i}(\bar{b'})$, and in a counterfactual way the components $m_{1i}(\bar{a'})$ and $m_{2i}(\bar{b})$, $i = N + 1, \ldots, 2N$. Similarly, on the third ensemble, $E^{(3)}$, one measures actually $m_{1i}(\bar{a'})$ and $m_{2i}(\bar{b})$, and defines counterfactually the components $m_{1i}(\bar{a})$ and $m_{2i}(\bar{b'})$, $i = 2N + 1, \ldots, 3N$. Finally, on the fourth ensemble, $E^{(4)}$, one measures actually $m_{1i}(\bar{a'})$ and $m_{2i}(\bar{b'})$, and defines counterfactually the components $m_{1i}(\bar{a})$ and $m_{2i}(\bar{b})$, $i = 3N + 1, \ldots, 4N$.

Consider now the global ensemble $i = 1, \ldots, 4N$, consisting of the four subensembles $E^{(1)}$, ..., $E^{(4)}$. For each individual element of this global ensemble a quadruple of values $\Big(m_{1i}(\bar{a}), m_{1i}(\bar{a'}), m_{2i}(\bar{b}), m_{2i}(\bar{b'})\Big)$ may be defined (two actual results and two hypothetical, yet physically relevant, results). We are now back at the stage in the first derivation of the BI where for each quadruple the inequality (80) applies, which leads again to the BI (81).

All this may still be reformulated in the following equivalent way: the justified use of counterfactually assigned values in CM (based on the validity of RH in CM) amounts to the justified introduction in CM of a "joint measurement" distribution or jpd for any kind of observables, in the previous case for four angular momentum components. The assumption of the existence of a joint measurement distribution for these components, gives again rise[38, 37, 39] to the physically relevant BI (81).

Conclusion

Our results may be summarized as follows. For the empirical verification of the BI in EPR experiments (i.e. those in which only two observables are measured actually), real actual correlation functions $C(\bar{a}, \bar{b})$, etc. should be filled in. This implies that, in order to retain the physical relevance of the BI in those experiments, it must be possible to actually realize the counterfactual, hypothetical, individual situations used in its derivation. Within the domain of classical physics this requirement is evidently met. It may be concluded that, if in some framework the validity of locality is taken for granted from the outset (e.g. because it is in accord with all our observations), the remaining issue is the reproducibility of individual physical states.

7.4. The use of CFD in quantum physics

In sect. 7.2. we have seen that the physical situations for which valid CFD argumentations may be set up are those for which a theoretical framework allows for *deterministic* predictions, given the state function which represents the situation. As is known, in the domain of application of QM deterministic predictions can only be made for *frequencies* of outcomes within ensembles. It follows that in QM counterfactual reasoning is justified only when applied to ensembles.

Deterministic predictions for frequencies within ensembles may be transformed to probabilistic predictions for single outcomes. According to these two different ways of formulating predictions in QM, a classification of interpretations may be made according to whether the formalism is assumed to apply to individual quantum systems ("single–system interpretations") or to ensembles of such systems ("ensemble–interpretations"[4]). To the first class belong a variety of interpretations which go under the name of Copenhagen interpretation of QM (CIQM),[61] as well as the one used by EPR[8] in their criticism of Bohr's claim of the completeness of QM. This CIQM is advocated by Bohr and Heisenberg: Bohr[9] argued for the completeness of QM within such an interpretation, and Heisenberg[62] attached to quantum systems a property of potentiality as an explanation for the stochastic (probabilistic) character of individual quantum events. As will be explained below in terms of CFD considerations, this group of (single–system) interpretations has been criticized by EPR and should be discarded because it leads to contradictory conclusions (known as the "EPR paradox").

Because there does not exist a unique satisfactory interpretation of QM, the problem of the justified use of CFD in the domain of QM, is more complex in QM than in CM. However, introducing again the requirement of empirical verifiability of theoretical predictions may clarify the situation. It is because this criterion is mostly overlooked, that these two broad kinds of formulating predictions in QM have given rise to much confusion with respect to the justified application of CFD within the quantum formalism. Applying this criterion consistently, only *deterministic* predictions are valid ones, i.e. in the quantum case only interpretations of the ensemble–type are physically acceptable. The criterion of empirical verifiability can be applied to quantum mechanics most naturally in the empiricist interpretation (cf. sect. 2.2.), since, strictly speaking, only the macroscopic phenomena related with preparation and measurement are liable to it. Since at this moment we do not have complete control over the individual preparation event, it is clear that an empiricist interpretation can only be compatible with ensemble–type considerations. From this point of view the EPR experiment is neither paradoxical nor a problem, since the relative frequencies of its measurement outcomes are described perfectly well by quantum mechanics.[63] As already remarked in sect. 2.2., in the empiricist interpretation unperformed experiments have no results. Hence, the problem of counterfactuals does not arise.

As discussed in sect. 5.1. the empiricist interpretation leaves unanswered certain questions about causal relations between events. According to van Fraassen[64] this is no drawback, because "the conviction that in science all persistent correlations must be explained" can be denied. Although from an empiricist point of view this is a perfectly respectable standpoint, we think that, methodologically, the causality question is an important one, not to be abandoned too hastily. As a matter of fact, as is clear from the possibility of extending the domain of application of quantum mechanical theory to experiments described by POVM's (cf. sections 3. and 4.), there is no immutable empirical regime. What was metaphysical yesterday (like the joint measurement of incompatible observables) may be empirical today. Even though also in the extended theory no complete causal explanation is given of EPR correlations, it is strongly suggested that the influence of the measurement arrangement must be included among the (partial) causes. As discussed in sect. 2.3. much confusion about the meaning of the uncertainty relations is a consequence of the failure to distinguish the different (partial) causes to measurement uncertainty due to preparation and measurement.

Hidden–variables theories have their origin in a desire to give a causal description of individual systems rather than ensembles. At this moment such descriptions are metaphysical in that they are not verifiable in practice. A rejection of causal explanation is tantamount to saying that they are unverifiable even *in principle*. Indeed, if one would take seriously both nonlocality as a necessary consequence of hidden–variables descriptions and the empirical locality of the quantum mechanical statistical measurement results, then a rejection of hidden–variables descriptions on the basis of their metaphysical character would be the more reasonable one.

Yet, as discussed in sect. 5., certain quantum mechanical facts, –among them the EPR correlations– , seem to ask for a more in–depth explanation, the assumption of a common cause being the most plausible one for the explanation of correlations. It is important to notice that the notion of a common cause is not incompatible with the uncontrollability of individual preparations mentioned earlier. Indeed, in certain experimental situations (like the singlet state to be discussed in the sequel of this section) it is possible to obtain unambiguous information on one individual particle by measuring an observable on the other (correlated) particle. Uncontrollability does not matter here.

These points will be further discussed below by investigating the crucial dependence of the validity of the argumentations of the Bell and of the EPR type on the justified use of CFD in their respective context.

7.4.1 The use of CFD in Bell–type argumentations

In his re–investigation of the EPR reasoning Bell tried to answer the question whether a HV scheme reproducing the predictions of QM in its domain of validity, could still incorporate the idea of local interactions. His negative answer constitutes the essence of Bell's theorem: "no local HV theory can reproduce *all* the results of QM". As is known, subsequent experimental results concerning Bell's theorem have been interpreted as "unambiguous" evidence for the property of the nonlocality of QM. In many papers this conclusion is even termed "unavoidable". However, it will be argued below that, despite the existing consensus, such a conclusion is largely exaggerated, and that, instead, it is the justified use of CFD or of the underlying physical property RH which is at stake.

In our further discussion we will make use of the criterion of verifiability of the predictions and apply this consistently in investigating whether from the violation of the BI by QM the nonlocality of QM follows necessarily (Bell's theorem). The application of this criterion then implies that the only meaningful question to be answered is: "do the *frequencies* depend nonlocally on measurements performed in spacelike regions?" Such a question belongs to the domain of applicability of QM and the unambiguous answer is: the frequencies are independent of such measurements,[63, 65] and, hence, QM is a local theory in this respect. It is the apparent contradiction between observable locality in a statistical sense and unobservable nonlocality in an individual sense, which we will try to resolve below.

It is clear that the issue of Bell's study was not this statistical locality. Instead, it concerned the question whether a more detailed theory for the individual case, coinciding with QM predictions in the quantum domain, could still have locality among its basic properties. Bell's considerations were based on the notion of hidden variables. It is sometimes claimed that in deriving the BI for EPR experiments one can do without introducing hidden variables. However, as explained in sect. 7.1., such a claim is not tenable because quantitative considerations on individual cases necessitate the introduction of a new, more general, formalism aimed to correspond e.g. with *more* sophistication with respect to our control over physical situations. It is clear that such an enhanced control cannot be reflected in the formalism by the *same* number of variables (as in QM), but only by the introduction of *more* variables. Because the HV program is usually defined as the embedding of QM within a classical–like sub–quantum theory[59] we have used in section 7.3. Stapp's[60] argumentation for deriving a Bell–type inequality for a formalism of the classical type. Stapp's approach is chosen because in it CFD is used in an explicit way. It consists in combining explicitly one single actual measurement result with three hypothetical ones which are considered to be physically meaningful on account of the *explicitly* assumed validity of CFD. However, in sect. 7.3. it has been explained that the physical relevance of the whole procedure depends on the *justified* use of CFD, which itself rests on the validity of a reproducibility condition RH at the *individual* level.

The hypothetical HV formalism concerns events in the *quantum* domain for which it is known that the classical formalism fails. We have seen that one major difference between the classical and the quantum formalism is a restriction as to the compatibility of observables. For the same reason it is not a surprise that a HV formalism that is modelled after the classical formalism, leads to an inequality, the BI (81), which is violated by the quantum formalism and by experiment. Because the proposed HV formalism concerns *individual* quantum events, it is clear that some fundamental *classical* property has to be invalid. Now, because of the empirical validity of LOC, even between causally related *individual* quantum events (as can be verified e.g. in EPR experiments measuring strictly correlated quantities), it is the alternative possibility, namely the justified use of CFD which is at stake. As we have seen above, CFD is linked to the property of reproducibility of sub–quantum states. Our conclusion is that, although physics concerns, up to now, reproducible phenomena, the validity of *reproducibility* of *individual* physical states may break down at the level of description of individual quantum events.

In recent years, Stapp too has become aware of the crucial importance of CFD in the locality issue[66]: "The key operative concept of the EPR–Bell argument is ... counterfactual definiteness.", and according to him "No satisfactory derivation of nonlocality, or the existence of faster–than–light influences, can be based upon such a CFD assumption: a failure of this assumption is (at least in my [i.e. Stapp's] opinion) far more likely than the existence of a faster–than–light influence." As a consequence, Stapp has tried to set up an argumentation *without the explicit use* of CFD[66] which, if correct, would make the conclusion of nonlocality inescapable. However, it has been shown by the present authors[67] that Stapp's attempt is deficient in that his reasoning still relies on the now *implicit* use of CFD in an unjustified way. In order to make this clear we repeat here Stapp's "Unique results" assumption, which he concedes[68] to be crucial in his derivation of the BI. According to this assumption "For each of the alternative possible (EPR) measurements $m, m = 1, \ldots, 4$ if m is performed then nature will select some unique value for the result of this experiment, and will never fix any value for the results which the remaining three measurements would have had if they had been performed." As it stands this is a rather innocent assumption, completely in accord with an empiricist interpretation of quantum mechanics. As applied by Stapp, however, the assumption acquires a highly nontrivial significance. In Stapp's opinion[68] it must be possible to "contemplate, in the situation created by one single preparation, *alternatively*, each of the four *alternative* possible future measurements."

It is impossible for us *not* to interpret this as an assumption of the *possibility of identical initial states* in the different EPR experiments. Such an assumption may seem harmless from the usual point of view of quantum mechanics, in which in different EPR experiments the same state function can be taken as the initial one, which, then, would seem to imply *identical preparation* in these experiments. However, as discussed in earlier sections we have reasons to doubt this interpretation of the quantum mechanical formalism. Indeed, it is one of the objectives of HV theories to distinguish between different individual preparations by means of the introduction of hidden variables, and condition the measurement results on the initial values of these variables (sect. 6.). It seems to us that Stapp only wishes to condition on the quantum mechanical state function, which, in the orthodox interpretation of quantum mechanics, "must refer to the single whole macroscopic preparation".[(68)]

By taking the possibility of identical initial states for granted Stapp remains within the confines of the orthodox interpretation of quantum mechanics. This very interpretation is an additional assumption in his derivation of the BI. Only if this interpretation would be the unquestionably true one, Stapp's reasoning would imply that a violation of LOC can be inferred from an experimental violation of the BI. Hence Stapp's reasoning does not bring us any further than the conclusion already drawn by Einstein[(10)] that if quantum mechanics is complete (and hence does not allow to distinguish between preparations that are "identical" in the orthodox sense), then it must be nonlocal (cf. sect. 2.1.). However, even now we are not compelled to accept the orthodox completeness dogma any more than Einstein was. As long as all experimental evidence remains at variance with nonlocality, it would be wise to doubt any assumption having nonlocality as a consequence. From this perspective not locality but the extra assumption of identical preparation in different EPR experiments is the dubious one. If we would maintain locality it is even possible to interpret Stapp's proof as an indication that it is not allowed to assume that identical preparations are possible in different (incompatible) EPR experiments, i.e., our NRH hypothesis.

By accepting the possibility that one and the same preparation can be combined with different EPR experiments Stapp, through his "unique results" assumption and a subsequent appeal to locality, arrives at a quadruple of measurement results. From this the BI is derived, much in the same way as this can be done from our jpd (69), in which, as mentioned in sect. 6.3., the assumption that the same initial HV state can be assumed in the four EPR experiments is crucial. By this latter assumption it is possible to contemplate conditional probabilities for measurement outcomes of different EPR experiments conditioned on the same value of λ, which, in the deterministic case, yields well–defined outcomes for all observables. This is tantamount to counterfactual definiteness. Of course, Stapp does not consider explicitly hidden variables, but this seems to be only a semantic question, Stapp insisting on calling "a single initial preparation" what is usually indicated as a sequence of "identical" preparations (ensemble interpretation of quantum mechanics) or even a sequence of ontologically different preparations (HV theories). The role played by Stapp's labelling of individual pairs of particles is not essentially different from our labelling by the HV λ. Therefore also our conclusion drawn above with respect to CFD is applicable. It seems to us that by employing the obscuring language of the orthodox interpretation Stapp is cut off from the possibility of seeing an essential assumption that is at the basis of his "derivation" of quantum mechanical nonlocality, an assumption that is neither necessary nor plausible. The violation of the BI by QM may thus be explained by the violation of the *reproducibility* property of individual physical situations[(58, 69)] in the domain of quantum processes.

7.4.2 The use of CFD in the EPR argumentation

Although it is common practice to discuss Bell's theorem as an application of the EPR problem, there is a fundamental difference between the experimental situations referring to these two, the difference consisting in the fact that in the second one *no* measurement is performed on one of the two correlated particles, thus preventing a strictly empiricist analysis. Therefore, from this latter point of view the EPR problem does not have any meaning, and Bohr's answer can be understood only on the basis of certain realist elements in his interpretation of the quantum formalism (cf. sect. 5.). On the other hand, an objectivistic realist interpretation in which quantum mechanics can be seen as a description of an *objective* reality was EPR's explicit point of departure in defining the "element of physical reality" according to the well–known statement: "If, without in any way disturbing a system, we can predict with certainty (i.e. with probability unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." It is precisely this realist interpretation, quantum mechanics not describing the results of actually performed measurements but describing the microscopic object itself, that may be responsible for the blurring of the distinction between the EPR and the Bell case. As suggested above, this difference is most striking from the empiricist point of view because, contrary to the *physical* Bell problem (dealing specifically with *observed* quantities), the EPR problem could simply be ignored as metaphysical (dealing with both *observed* and *unobserved* quantities). Therefore, our investigation below of the implicit dependence on CFD of EPR's original incompleteness argumentation shall be carried through in the objectivistic realist interpretation.

The idea that quantum mechanics yields an objective description of reality is at the basis of the application of CFD in the EPR reasoning: if a physical quantity is an objective property of the object, and if, without in any way interacting with the object, the value of this physical quantity can be predicted with certainty by some measurement, then the quantity must have had this value before the measurement, and independently of any measurement to be performed afterwards. According to the usual axioms of quantum mechanics, prediction with certainty is only possible when the object is in an eigenstate of the measured observable. Hence, in the objectivistic realist interpretation, the object must have been in the corresponding eigenstate already before the measurement. This must be compared with the orthodox interpretation in which no value of any observable can be attributed to the object before the measurement.

The EPR argumentation had the intention to challenge Bohr's provocative claim that quantum mechanics is a complete theory, i.e., that the quantum formalism describes the system in an exhaustive, i.e. complete, way. In particular, in a realist interpretation of the state function this could be interpreted in the sense that in a specific preparation a single ontological physical situation was created which was exhaustively represented by a quantummechanical state vector, meaning that no more detailed specification of this situation was possible. It was EPR's intention to show that such a more detailed specification is possible in certain physical situations, but that quantum mechanics is not able to implement this possibility.

In Bohm's version of EPR a single system is prepared in the singlet–state $|00\rangle$ and decays in two subsystems S_1 and S_2 of spin 1/2, which from that moment on do no longer interact. The spin components of system S_1 may then be measured along one of two different directions, $\bar{a}$ or $\bar{b}$. If a measurement along direction $\bar{a}$ is planned, then one may write:

$$|00\rangle = \frac{1}{\sqrt{2}} \left[\left| u^1_{\bar{a}} \right\rangle \left| u^2_{-\bar{a}} \right\rangle - \left| u^1_{-\bar{a}} \right\rangle \left| u^2_{\bar{a}} \right\rangle \right] . \tag{82}$$

This relation is then interpreted in the following way: if the result for S_1 is $m^1_{\bar{a}} = +1$, then the result $m^2_{\bar{a}} = -1$ is predicted, and the corresponding state $\left| u^2_{-\bar{a}} \right\rangle$; if the result for S_1 is $m^1_{\bar{a}} = -1$, then the result $m^2_{\bar{a}} = +1$ is predicted, and the corresponding state $|u^2_{\bar{a}}\rangle$.

In an objectivistic realist interpretation of quantum mechanics the state function $|00\rangle$ is thought to yield an objective description of the reality of the prepared system, independently of the measurement that is to be performed. This, then, justifies EPR to consider counterfactually the idea "If, instead of this [i.e. a physical quantity A], we had chosen another quantity, say B, ... we should have obtained, instead of [eq. (82)] ..., the expansion [(83) see below] ...". The new quantity B may represent the measurement on S_1 along direction $\bar{b}$ (assumed not collinear with $\bar{a}$) and the corresponding relation is now

$$|00\rangle = \frac{1}{\sqrt{2}} \left[\left| u^1_{\bar{b}} \right\rangle \left| u^2_{-\bar{b}} \right\rangle - \left| u^1_{-\bar{b}} \right\rangle \left| u^2_{\bar{b}} \right\rangle \right] , \tag{83}$$

from which a similar conclusion as above may be inferred concerning the state of S_2. Thus, if quantum mechanics would be a complete theory, the *same* system S_2 would have to be in two completely different states (each of them supposed to represent the *real* state of the system, in the same way as this is thought to be the case in classical physics) although, by assumption, no interaction can be responsible for the creation of this state. EPR's contradictory conclusion, then, is that "*... it is possible to assign two different wave functions ... to the same reality* (the second system after the interaction with the first)."

The two basic assumptions leading to the contradiction are i) CFD, ii) completeness of quantum mechanics. The contradiction can be removed by abolishing one of these assumptions. For EPR the first assumption was beyond question: according to EPR "... this criterion (i.e. the reality criterion as expressed by the element of physical reality) is in agreement with classical as well as quantum–mechanical ideas of reality". Since the reality of S_2 is not altered by a measurement on S_1, the spin components of S_2 in both directions must be well–defined before any measurement is done actually measuring such a component. Hence, they must be counterfactually true. From the contradictory result obtained above they conclude that CFD of incompatible quantities cannot be implemented in the quantum mechanical formalism, and that, hence, quantum mechanics must be incomplete.

The agreement with classical ideas is clear, as follows from our discussion on the use of CFD in the classical case (section 7.3.). However, as far as quantum ideas are concerned, this assertion ignores the possibilities of both contextualistic realist and empiricist interpretations of the quantum formalism. As is well known, Bohr did not entertain a realist interpretation of the state function. As is discussed in sect. 5. Bohr, however, did entertain a realist interpretation of observables. It, presumably, is not a coincidence that both the EPR completeness criterion ("Every element of physical reality must have a counterpart in the physical theory") and Bohr's answer[9] are phrased in the language of observables rather than states: only the language of observables provided sufficient common ground to be able to express disagreement. From Bohr's answer[9] it is rather clear that he considered CFD to be the assumption on which the EPR reasoning is unjustifiedly based (cf. sect. 2.1.): according to him the element of physical reality cannot be an objective one, but is defined only within the context of an experimental arrangement (this possibility was already mentioned in the EPR paper itself). Hence, there is no point in the assumption of an element of physical reality for B in the context of an A measurement if A and B are incompatible. According to Bohr the EPR element of physical reality is ambiguous because it is defined independently of the measurement arrangement. In Bohr's view quantum mechanics describes reality only in as far as it is interacting with (or related to (cf. sect. 2.2.)) a measuring instrument. Quantum mechanics is complete because it is impossible to avoid the disturbing influence of this interaction.

It was Einstein's own favorite conclusion that QM describes at best an *ensemble* of similarly prepared systems, and should be completed by a more powerful theory for the single case. It is important to note that this kind of completion can hardly have been the one that Bohr had in mind. Bohr must have been aware of the fact that quantum mechanics only yields *statistical* predictions, and that it is incomplete in this sense. Bohr never denied the incompleteness of quantum mechanics in this respect. Bohr's completeness thesis must be understood in the sense that the quantum mechanical description cannot be improved on, whether or not it is possible to contemplate an underlying physical reality in each individual measurement act (as a matter of fact Bohr's thinking was rather a realist one in this respect.[17]) Bohr never separated the completeness issue from the issue of complementarity, to the effect that it is our fundamental inability to obtain knowledge about a microscopic system *without interacting with the object* that is responsible for the completeness idea, rather than the impossibility of contemplating the object as being prepared in a well–defined initial state.

This is another reason for making a clear distinction between the EPR and the Bell case, the latter being originally intended to implement the Einsteinian idea of completion by means of hidden variables. By itself this is a completely logical development because in a HV theory CFD can be understood in terms of the value the HV λ had before the measurement. However, the very assumption of CFD is contrary to the Bohrian way of thinking, not so much because of its realist implications as well as because of its independence of the measurement arrangement, i.e., its objectivity. For Bohr the state function did not have the realist meaning of the description of an initially prepared state as it had for Einstein. Therefore Bohr could not reject Einstein's assumption of equal initial state preparations in different experiments. However, Bohr rejected EPR's assumption of the same element of physical reality for system S_2 in different experiments. By Einstein[10] this was interpreted as a consequence of nonlocality (on the assumption of completeness, cf. sect. 2.1.). As far as Bohr shares a certain realism with Einstein, however, it can also, and alternatively, be interpreted as the impossibility of preparing the same ontological state in different experiments to be performed on S_2. This latter interpretation would coincide with our hypothesis of non-reproducibility (NRH), which, for this reason, could avert the necessity of Einstein's nonlocality conclusion, without the obligation of rejecting completeness as inherent in Bohr's complementarity idea.

7.4.3 A possible explanation of nonreproducibility in quantum mechanics

De Broglie's idea [(70)] of quantum mechanics as describing microscopic objects interacting with a "hidden thermostat" may offer a possible explanation of the nonreproducibility of individual ontological states, by exploiting the analogy with the situation obtaining in thermodynamics. A thermodynamical state is a state of (local) thermal equilibrium in which deterministic behaviour at the macroscopic level is accompanied by chaotic behaviour at the microscopic level. If quantum mechanical states would be *equilibrium states* of a subquantum dynamics, the deterministic evolution of the wave function would, therefore, be comparable with the analogous evolution of solutions of the heat equation. Reproducibility of an individual ontological state $|\lambda)$ would have to be compared with reproduction of the exact initial state of 10^{23} classical particles. For this reason reproduction of such a state would seem to be a factual impossibility.

There is yet another aspect of the nonreproducibility hypothesis that may even be more important than the factual nonreproducibility mentioned above. An important property of equilibrium is its contextual character. Equilibrium states of thermodynamics are dependent on the experimental context. For instance, the macroscopic state of a volume of gas evidently is dependent on shape and orientation of the container. If quantum mechanics would be a theory about equilibrium states of a subquantum dynamics, it is to be expected that these equilibrium states will be in equilibrium with the measuring equipment. Hence, mutually excluding measurement arrangements could not accommodate the same equilibrium state. In this way nonreproducibility would be directly related to one of the most basic characteristics of quantum mechanics, viz., complementarity.

One final remark should be made with respect to the representation of quantum mechanical probability distributions by (64) or (66). In these expressions a role is played by conditional probabilities like $p(a_k|\lambda)$, conditioned on an individual ontic state ($|\lambda)$. This would mean that these probabilities are physically determined by an *instantaneous* state of the hidden variables. If the thermodynamical analogy is valid, this is highly questionable. As a matter of fact, also thermodynamical properties like temperature and pressure are not instantaneous properties of the object but correspond with quantities averaged over certain time and space regions: they are conditioned on states of (local) equilibrium. It is not unreasonable to surmise that also quantum mechanical measurements do not probe the instantaneous hv state $|\lambda)$, but yield only information on the equilibrium state. This would imply that conditional probabilities like the ones used in (64) and (66) have no physical relevance within the domain of quantum mechanics. This would make obsolete their use in derivations of the Bell inequalities as following from (69), if these are to be applied in quantum mechanical measurements: conditional probabilities for results of incompatible observables could not be conditioned on the same equilibrium state. Elaboration on these ideas is relegated to a future publication.

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