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Can We Escape from Bell's Conclusion that Quantum Mechanics Describes a Non-Local Reality? ⋆

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It is argued that for a proper understanding of the question of non-locality in quantum mechanics and hidden variables theories purporting to reproduce the quantum mechanical measurement results, it is essential to consider stochastic hidden variables theories. Laudisa's (1996) conclusion that in derivations of the Bell inequality an implicit assumption of locality is made, is shown to be a consequence of his restriction to deterministic hidden variables theories. It is also demonstrated how it is possible to draw a clear distinction between contextualism and non-objectivism, non-objectivism amounting to the impossibility of reducing an individual quantum mechanical measurement result, either in a deterministic or in a stochastic way, to the hidden variables state the individual object is in independently of the measurement. The analogy with thermodynamics is exploited to clarify the issue. Copyright ©1996 Elsevier Science Ltd.

1. Introduction

When John Bell derived his inequality his inspiration was two-fold: (i) his conviction that von Neumann's proof of the impossibility of hidden variables failed; and (ii) his conviction that Bohm's theory provided an example of a hidden variables theory that was observationally equivalent to quantum mechanics. The non-locality of Bohm's theory gave him the idea that, if hidden variables were to be proven impossible at all, this should be restricted to *local* hidden variables theories. For this reason he restricted his attention to these latter theories, and, ever after succeeding in deriving his inequality, was convinced that the locality assumption was essential to the derivation. Local hidden variables theories thus having been demonstrated to be incompatible with the (empirically

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corroborated) results of quantum mechanics, it is widely believed that the quantum world must be a non-local one. In so-called EPR experiments, in which two observables A and B are measured jointly in causally disconnected regions of space–time, this would mean that an individual measurement result of A (e. g. a_i) in one region would (also) depend on the measurement arrangement (B) chosen in the other region. Because of the causal disconnectedness of the regions this would imply a non-local influence exerted by B on the measurement of A. We will refer to this as Bell non-locality, symbolized by the inequality:

$$a_i(A, B) \neq a_i(A, B'), \;\; B \neq B', \tag{1}$$

in which a possible dependence of the measurement result on the measurement arrangement [either (A, B) or (A, B')] is shown explicitly.

On the other hand, the quantum mechanical postulate of local commutativity tells us that A and B should commute, thus implying the independence of the probability distribution of one observable from the choice of the other one. Thus, if $p_{AB}(a_i, b_k)$ is the joint probability distribution obtained in the (A, B) measurement, then

$$\sum_k p_{AB}(a_i, b_k) = \sum_l p_{AB'}(a_i, b'_l) = p_A(a_i). \tag{2}$$

This is indicated as *statistical locality* by Laudisa (1996, Section 3). A coexistence of Bell non-locality and statistical locality has sometimes been felt as rather unsettling. Thus, Herbert (1988, p. 161) characterizes it as an 'uneasy truce'. According to Laudisa the coexistence of non-locality in the sense of Bell with statistical locality is possible and well understood. This understanding is illustrated by alluding to Bohm's causal interpretation of quantum mechanics, allegedly combining the statistical results of quantum mechanics with a non-local individual particle dynamics.

In assessing the question of the coexistence of non-locality on the individual level with locality on the statistical level, the key problem is why experiment would be hiding on the statistical level the effects of a violation of locality exerted on the individual level. If Bohm's theory were a genuine hidden variables theory, offering an explanation of the empirical phenomena described by quantum mechanics by means of a *non-local* causal description of the individual object, then it would seem to me that the unobservability of non-locality would be one of the foremost characteristics to be explained. The merits of Bohm's theory as an explanation of quantum mechanical phenomena were reviewed in de Muynck (1987). This will not be repeated here. Suffice it to say that Bohm's theory is observationally equivalent to quantum mechanics. The causal mechanism in Bohm's theory is added to the quantum mechanical formalism as a piece of extra interpretation, not providing any extra observational evidence over that described by the quantum mechanical formalism itself.[1] For this reason any consequence following from this extra interpretation can be qualified as

[1] For this reason I welcome Laudisa's terminology, referring to 'Bohm's causal interpretation of quantum mechanics' rather than to 'Bohm's hidden variables theory'.

metaphysical. In particular, the non-locality of the theory can be qualified as such since all experimentally observable phenomena known up until now satisfy statistical locality. This could hardly be expected if the underlying mechanism were non-local. Rather than being an explanation of quantum mechanics, the causal mechanism is itself in need of an explanation, viz why no single trace of the non-locality of the causal mechanism can be found experimentally.

To say that the coexistence between Bell non-locality and statistical locality is well understood is tantamount to having an answer to this question. As far as I know, however, such an answer has never been given, and is also not to be found in the article by Laudisa. It seems to me that such a conspiratorial behaviour of a non-local quantum reality, hiding its non-locality from observation by means of quantum mechanical measurements, is implausible, unless physical reasons can be given. There is, however, no single indication that such a physical explanation can be found. Bell non-locality has properties reminiscent of the strange properties of the world aether, and is bound to be rejected for analogous reasons. As long as (2) is found to be experimentally satisfied, the truce between Bell non-locality and statistical locality, although logically possible, is hardly physically plausible. The availability of Bohm's theory may be for Laudisa, as it was for Bell, instrumental in underestimating the fundamental difficulty in reconciling statistical locality with Bell non-locality.

2. Non-Objectivism and Contextualism

Laudisa's analysis of the subject of non-objectivism and contextualism is largely based on a paper (de Muynck, 1983) intended to demonstrate that so-called *objective*[2] local hidden variables theories are incapable of reproducing quantum mechanical probabilities. Laudisa rightly notes that at that time, and maybe even at this moment, these notions are not uniquely defined in the literature. One purpose of the present paper is to try to contribute to their clarification, and to show in what sense I think that they are important in understanding the meaning of quantum mechanics.

As observed correctly by Laudisa, in many discussions the notions of non-objectivity and contextuality are used more or less synonymously (his Interpretation 1). This is caused by the fact that quantum mechanics is often interpreted as a theory of *measurement*, preparation being treated as unproblematic. The question is here whether quantum mechanical observables can be considered as *objective* properties, to be attributed to the object system independently of any measurement, measurement only revealing the value the observable objectively possessed beforehand. It is one of the central tenets of the Copenhagen interpretation that such an interpretation is impossible, and that observables only can have their values in the context of the measurement of this very observable. Nowadays a rather strong tendency can be observed to follow Ballentine (1970) in his statistical interpretation of quantum mechanics, in which measurement is

[2] In the sense defined by Clauser and Horne (1974).

denied a fundamental position among physical processes, and the Heisenberg uncertainty relations are interpreted as restrictions on our ability to perform *preparations* rather than measurements. This subject has been recently discussed extensively by de Muynck *et al.* (1994). There does not seem to be any difference with Laudisa with respect to our conclusion that an objectivistic interpretation of quantum mechanics in this sense would fail to account for the possibility of a violation of the Bell inequalities. We seem to agree on the fact that the measurement procedure must play an important role in the interpretation of quantum mechanics.

Let us now turn to hidden variables theories. It is clear that if we do not ask more from the hidden variables theory than a reproduction of the results of quantum mechanics, then we might analyze such a theory using the same categories as the ones applied to quantum mechanics. As discussed in Section 1, however, we do require more from our hidden variables theories. And we have the possibility to do so, because, contrary to the quantum mechanical description, now we have the possibility of describing the *individual* preparation by specifying the value of the hidden variable λ. This opens up the possibility of distinguishing between the notions of non-objectivity and contextuality.

In hidden variables theories quantum mechanical measurements (i.e. measurements falling within the domain of application of quantum mechanics) are often characterized by the quantities $p(a_i|\lambda)$ representing the conditional probabilities of measurement results a_i of the quantum mechanical observable A if the object is prepared with hidden variable value λ. Then, if $\rho(\lambda)$ is the relative frequency of the hidden variable as prepared in the initial state, the quantum mechanical probability $p(a_i)$ is given by the expression

$$p(a_i) = \int_{\Lambda} p(a_i|\lambda)\ \rho(\lambda)\ \mathrm{d}\lambda, \tag{3}$$

Λ being the hidden variables space. For so-called *deterministic* hidden variables theories the conditional probability $p(a_i|\lambda)$ reduces to the characteristic function of the region Λ_{a_i} of the hidden variables space Λ for which $A(\lambda) = a_i$. Since the Bell inequalities can be derived just as easily for *stochastic* theories as for deterministic ones, we shall in the following always deal with the former theories, deterministic theories forming just a special case. This has the advantage of exhibiting more clearly the fundamental difference between the quantum mechanical observable A and the hidden variable λ that will play an important role in the non-objectivity question. Whereas the latter can be thought of as an objective property of the object system (more or less analogously to classical mechanics), this clearly need not be true of the quantum mechanical observable, since the conditional probability $p(a_i|\lambda)$ may depend on the measurement arrangement (in the deterministic case this dependence would obtain for the region Λ_{a_i}). This can be made manifest by explicitly indicating the measurement arrangement. Thus, $p_{AB}(a_i|\lambda)$ is the conditional probability of a_i in the context of an (A, B) measurement. For stochastic theories the question of Bell non-locality boils down to the inequality

$$p_{AB}(a_i|\lambda) \neq p_{AB'}(a_i|\lambda), \tag{4}$$

replacing the inequality (1).

Hidden variables theories for which the conditional probabilities depend on the measurement arrangement are usually called contextual ones. As Laudisa (1996, Section 2) points out correctly, also for local contextual hidden variables theories in which (3) is assumed as a representation of quantum mechanical probabilities, the Bell inequality can be derived. Therefore such theories cannot reproduce the quantum mechanical measurement results. Does this undercut, as Laudisa states, the possibility of reproducing quantum mechanical predictions by a local contextual hidden variables theory? This would be true if hidden variables theories of the kind discussed above were the most general ones, thus making the Bell inequalities derivable in any such theory. However, hidden variables theories yielding (3) as a representation of quantum mechanical probabilities need not be the most general ones. Such theories were termed quasi-objectivistic theories in de Muynck and van Stekelenborg (1988) because of the possibility of interpreting the conditional probability $p(a_i|\lambda)$ as an instantaneous property of the object system. In Section 4 I will discuss the possibility that the assumption of the existence of the conditional probability $p(a_i|\lambda)$ may be too strong a requirement for quantum mechanical measurements, and should be relaxed. This implies the possibility of the existence of a class of local contextual hidden variables theories for which the Bell inequality cannot be derived, the theories not being objectivistic: neither in the strong sense that the observables themselves can be reduced to a value of the hidden variable, nor in the weaker sense of an attribution of the quantum mechanical probabilities via conditional probabilities $p(a_i|\lambda)$ (contextuality being implied by the dependence on the measurement arrangement).

As a matter of fact, Laudisa is discussing an analogous possibility in his Interpretation 2, viz Lochak's conjecture (Lochak, 1976; Lochak, 1977) that in (3) the relative frequency $\rho(\lambda)$ may depend on the measurement arrangement, i.e. $\rho(\lambda)$ is replaced by $\rho_A(\lambda)$. Since $\rho(\lambda)$ describes the preparation, contextualism now refers to preparation rather than to measurement. I agree with Laudisa that Lochak's model is not very plausible since it would imply that no objective preparation of the object system is possible, it evidently being determined already at the moment of the preparation which measurement is going to be performed. It is, indeed, not very likely that the possibility of deciding *after* the preparation which observable is going to be measured [as is intended in Aspect's switching experiment (Aspect *et al.*, 1982)] would be restricted by the laws of quantum mechanics.

Yet, Lochak's model is interesting because it draws attention away from measurement to preparation in quantum mechanics. It is, indeed, evident by now that in the Copenhagen interpretation the fundamental difference between preparation and measurement has been neglected, either by considering preparation and measurement as forming an indivisible whole, or even by interpreting preparation as a particular kind of measurement. Quantum mechanical preparation may be no less problematic than quantum mechanical measurement. Contextual hidden variables theories of type (3) seem to continue the Copenhagen tradition by only dealing with the question of measurement: quantum effects are thought to be stored in the conditional probabilities $p(a_i|\lambda)$ representing the measurement; preparation, represented by $\rho(\lambda)$, is treated classically. Since in such theories, even if contextual, the Bell inequalities are derivable, it is impossible that they reproduce quantum mechanics. Taking into account the influence of the measurement arrangement on the measurement (i.e. contextualism) does not seem to be sufficient for reproducing the structure of quantum mechanics by means of a hidden variables theory. In order to be able to account for quantum mechanical non-objectivity we will have to consider the preparation as well. I will return to this in Section 4, but want to address first Laudisa's Section 4 (his Section 3 having been dealt with in Section 1).

3. On the (Ir)relevance of the Bell Inequalities to the Problem of (Non-)Locality

Laudisa (1996, Section 4) asserts that in the derivation of the Bell inequalities there is an implicit assumption of locality, thus challenging de Muynck's opinion (de Muynck, 1986) that the Bell inequalities are irrelevant to the problem of (non-)locality in quantum mechanics. It seems to me that Laudisa's restriction in this section to *deterministic* hidden variables theories plays an important role in arriving at his judgment, because in such theories it is difficult to disentangle the notions of objectivity and locality. Thus, if a localized object has a value of a quantum mechanical observable as an objective property, then this property is necessarily a local one. Stochastic hidden variables theories evade this conclusion because in such theories a clear distinction can be drawn between measurement and preparation. Employing such theories it is less tempting to attribute quantum mechanical measurement results to the object as objective properties possessed by the object before the measurement.

In stochastic hidden variables theories the condition of locality is generally given as

$$p_{AB}(a_i, b_j|\lambda) = p_A(a_i|\lambda)p_B(b_j|\lambda). \tag{5}$$

Actually, the equality (5) is a condition of conditional statistical independence rather than a locality condition.[3] Physically it means that, given the preparation λ, the measurement processes of A and B do not influence each other in a statistical sense. This might be possible as well if the observables are measured in the same region of space–time, for instance, if they are compatible. Note that in *deterministic* hidden variables theories (5) simply means that the region of hidden variables space Λ for which both $A = a_i$ and $B = b_j$ is just the intersection $\Lambda_{a_i} \cap \Lambda_{b_j}$ and, hence, is not an expression of locality. It *is* an expression of mutual non-disturbance of the two measurement processes, though, but only in a

[3] Ballentine and Jarrett (1987) call it strong locality. It can easily be seen to be stronger than an assumption of Bell locality according to $p_{AB}(a_i|\lambda) = p_{AB'}(a_i|\lambda)$.

rudimentary sense since, due to the assumption of determinism, these processes are actually being ignored.

If the (A, B) measurement is an EPR measurement, A and B being measured in causally disconnected regions of space–time, then the mutual non-disturbance of the two measurement processes is warranted by the assumption of locality of the interactions between objects and measuring instruments. For this reason it is not unreasonable to refer to (5) as a locality property. The question to be discussed now is whether assumption (5) is essential to a derivation of the Bell inequality. This would be the case if the inequality cannot be derived without assumption (5). This, however, is not the case, because the Bell inequalities can be derived without this assumption. This follows from a theorem due to Rastall (1983) and Fine (1982a; 1982b), to the effect that satisfaction of the Bell inequalities is equivalent to the existence of a quadrivariate joint probability distribution $p(a_i, b_j, a'_k, b'_l)$ of the four observables A, B, A' and B' that are involved. Indeed, the locality condition (5) makes possible the explicit construction of such a joint probability distribution in the following way: let

$$p_{AB}(a_i, b_j) = \int_\Lambda p_A(a_i|\lambda)p_B(b_j|\lambda)\ \rho(\lambda)\ \mathrm{d}\lambda \tag{6}$$

be the bivariate probability distribution of the (A, B) measurement, and analogously for the other EPR experiments (A, B'), (A', B), and (A', B') (a possible dependence on the measurement arrangement is explicitly given so as to account for a possible contextuality, consistent with locality). It is now easily seen that the expression

$$p(a_i, b_j, a'_k, b'_l) = \int_\Lambda p_A(a_i|\lambda)p_B(b_j|\lambda)\ p_{A'}(a'_k|\lambda)\ p_{B'}(b'_l|\lambda)\ \rho(\lambda)\ \mathrm{d}\lambda \tag{7}$$

is a quadrivariate probability distribution, reproducing the bivariate EPR probabilities (6) as its marginals. This implies that, on these assumptions, the EPR experiments should satisfy the Bell inequalities.

It is clear that in the construction of this quadrivariate probability distribution use is made of the locality assumption (5), and rightly so because this is in accordance with the physical circumstances of the EPR experiments. This, however, does not imply that the locality assumption is necessary for a derivation of the Bell inequalities. According to the Rastall–Fine theorem these inequalities must be satisfied if a quadrivariate probability distribution exists. In the hidden variables theory this is fulfilled if conditional probabilities $p(a_i, b_j, a'_k, b'_l|\lambda)$ exist, such that

$$p(a_i, b_j, a'_k, b'_l) = \int_\Lambda p(a_i, b_j, a'_k, b'_l|\lambda)\ \rho(\lambda)\ \mathrm{d}\lambda. \tag{8}$$

It is completely unnecessary for a derivation of the Bell inequality that the conditional probability factorize according to (7).

In my view this clearly demonstrates the irrelevance of the locality assumption to the derivation of the Bell inequalities. From a physical point of view this might also be expected. The problem of quantum mechanics is connected with the existence of *in*compatible observables, corresponding to *non*-commuting operators, and their joint probability distributions. It is the joint measurement of *in*compatible observables rather than the joint measurement of compatible ones (as in EPR experiments) that is at the heart of the quantum mechanical conundrum. The quadrivariate probability distribution $p(a_i, b_j, a'_k, b'_l)$ can be interpreted as a result of a joint measurement of the four observables A, B, A' and B'. If these observables are all mutually compatible a quantum mechanical expression for $p(a_i, b_j, a'_k, b'_l)$ can be found, independently of whether the observables are measured in the same region or in causally disconnected regions, and the Bell inequalities must be satisfied. Violation of the Bell inequalities is only obtained if not all of the four observables are mutually compatible. This strongly suggests that violation of the Bell inequality is connected with *in*compatibility of observables. In hidden variables theories the Bell inequality is derived because the observables are treated classically, in the sense that they are treated (quasi-)objectivistically as properties the object has, either in a deterministic or in a stochastic sense, prior to measurement. Evidently, this is not allowed within the domain of quantum mechanics.

By itself this question of (non)-objectivity has nothing to do with the joint measurement of two compatible observables as considered in EPR experiments. Such measurements have, unfortunately, entered the discussion on the foundations of quantum mechanics only because it was thought to be impossible to measure jointly two *in*compatible observables, the latter seeming to me to be the real problem of quantum mechanics. The essential problem in all derivations of the Bell inequality is to arrange pairs of measurement results (a_i, b_j), (a'_k, b_j), (a_i, b'_l), (a'_k, b'_l), obtained in different (incompatible) EPR measurements, in quadruples (a_i, b_j, a'_k, b'_l), so as to be able to obtain a quadrivariate probability distribution from the relative frequency of these quadruples. There is no reason to believe that the impossibility to do so is caused by non-local influences in each of the separate EPR experiments. It is far more likely that it is the *combination* of results of *in*compatible measurements that causes the problems. This might have been already expected on the basis of the Heisenberg (1930, Chap. II) treatment of quantum measurement: since B and B' are incompatible observables, a measurement of B will disturb B'. This implies that, assuming the same individual preparation in the two EPR experiments (A, B) and (A, B'), the value b'_l of B' in the second measurement will differ from the one obtained in the first measurement. Hence, it follows that there is no unique value of b'_l for all EPR experiments, and it is impossible to form a quadruple. On this view, this impossibility, thwarting the derivability of the Bell inequality, is a direct consequence of one of the basic notions in the development of quantum mechanics, viz Heisenberg's disturbance theory of measurement, stating that a quantum mechanical measurement will disturb quantities that are incompatible with the measured one. Although also the inequality (1) prevents the existence of a unique quadruple (a_i, b_j, a'_k, b'_l), the assumption of Bell non-locality is not necessary to obtain this result.

I want to add here some remarks on possible reasons why this rather obvious explanation of the violation of the Bell inequality has not been generally accepted up until now, but has been superseded by the non-locality explanation. A first reason may be the fact that the non-locality issue already played a role in the discussion on the original Einstein–Podolsky–Rosen problem since Einstein's conclusion (Einstein, 1948) in his discussion with Bohr, that quantum mechanics must be non-local if it is thought to be complete. Hence, the Copenhagen completeness thesis of quantum mechanics may have been instrumental in people's acceptance of non-locality of the quantum world. On the other hand, Einstein's choice of locality, and, hence, of *in*completeness of quantum mechanics, was not generally accepted at that time, because it implied the 'metaphysical' assumption of the existence of hidden variables. Although Bohm and Bell have largely contributed to ridding hidden variables of their metaphysical character, they evidently were so much impressed by the non-locality of Bohm's theory that they concluded that also *in*completeness of quantum mechanics must be accompanied by non-locality.

A second reason may also stem from a certain fear of metaphysics. The above reasoning, demonstrating the impossibility of constructing quadruples of measurement results for the EPR experiments, hinges on the notion of joint measurement of incompatible observables. In particular it considers a value of B' if the incompatible observable B is measured instead. A strictly positivist attitude would deny the existence of a value of a quantum mechanical observable that is not actually measured. Indeed, for a long time, this has been the general attitude of the large majority of the physics community, such an attitude being promoted by the fact that in the Dirac–von Neumann axiomatization of quantum mechanics the joint measurement of incompatible observables does not have an obvious mathematical description (which is sometimes interpreted as an actual impossibility of measuring simultaneously or jointly incompatible observables). There, actually, is some tension between Heisenberg's disturbance theory of measurement, considering values of incompatible observables within the context of one single measurement arrangement, and the failure of Dirac–von Neumann quantum mechanics to account for these. This was already observed by Ballentine (1970), who concluded that (Dirac–von Neumann) quantum mechanics has nothing to say about the joint measurement of incompatible observables, and that, in particular, the Heisenberg uncertainty relations do not apply to joint measurements, but should be considered as restrictions on the preparation of the initial state, to be probed by means of separate measurements of the two observables involved.

Recently the above-mentioned tension between the *physics* of the joint measurement of incompatible observables and the quantum mechanical formalism has been relieved by the development of a generalized formalism in which quantum mechanical observables are represented by positive operator valued measures rather than by Hermitian operators (Davies, 1976; Holevo, 1982; Ludwig, 1983; Busch *et al.*, 1995). It was possible (Martens and de Muynck, 1990a; Martens and de Muynck, 1990b; Busch, 1987; de Muynck and Martens, 1990; Yamamoto *et al.*, 1990) to exploit the generalized formalism for describing the joint (non-ideal) measurement of incompatible observables, corroborating both the general ideas of Heisenberg's disturbance theory of measurement and Ballentine's conclusion that limitations on the mutual disturbance of incompatible observables are not to be described by Heisenberg's uncertainty relation but by a different one (Martens and de Muynck, 1990b, p. 366), following from the generalized formalism. Application of the generalized formalism to the problem of the Bell inequalities (de Muynck *et al.*, 1994) makes it possible to consider the joint measurement of all four observables A, B, A' and B' involved, and study the relation of such a measurement to the traditional EPR measurements. Violation of the Bell inequalities in these latter experiments can then be understood on the basis of Heisenberg disturbance, due to the fact that the measurement of B can be interpreted as a joint measurement of the incompatible observables B and B' (and analogously for A and A'), B' being maximally disturbed. No non-local influence by the other measurement (i.e. a measurement of either A or A', or a joint measurement of A and A') on the measurement results for B is necessary for such an understanding. There is nothing in the generalized quantum formalism violating (statistical) locality, independently of whether the Bell inequalities are satisfied or violated.

4. Non-Objectivistic Hidden Variables Theories

It is commented by Stapp (1994) that the analysis of the problem of the Bell inequalities in terms of the joint measurement of the four observables cannot be the whole story.[4] Put in this way, I fully agree with Stapp and Laudisa on this account, although for different reasons. Whereas Stapp thinks that joint measurements of the four observables are less incisive and less informative than the traditional EPR experiments (which is incorrect because the new experiments have been shown to yield at least the same information as the traditional EPR experiments), and cannot teach us more about non-locality than these traditional experiments themselves (which is doubtful because these latter experiments are just special cases of the new ones), it is my opinion that the analysis is necessarily incomplete because the treatment is a quantum mechanical one and, hence, is not able to describe the *individual* preparation. For this it will be necessary to rely on a hidden variables theory (a necessity denied by Stapp).

In Laudisa's opinion the generalized experiments are irrelevant to the problem of the Bell inequality because of a theorem also due to Fine (1982a; 1982b) that a local deterministic hidden variables theory exists if and only if the Bell inequality is satisfied. Hence, local determinism would be equivalent to satisfaction of the Bell inequality. It should be stressed here that the theory

[4] Actually, Stapp's formulation is much stronger, the new kinds of experiments being in his opinion basically irrelevant to the non-locality issue, a view shared by Laudisa (1996, Section 1).

describing generalized measurements is a purely quantum mechanical one, in which the Bell inequality can be studied independently of any assumption about the existence of hidden variables. As discussed in Section 3, this theory strongly suggests that a violation of the Bell inequality is tied up with the notion of a Heisenberg measurement disturbance in a joint (non-ideal) measurement of the four observables that are involved, a notion that can be implemented into the quantum mechanical formalism only within the generalized formalism (de Muynck *et al.*, 1994). The problem of the joint measurement of incompatible observables being at the centre of the enigma of quantum mechanics, the connection to this latter problem lends the Bell inequality an importance within quantum mechanics itself, independently of the question of hidden variables. Laudisa's judgment of irrelevance ignores the central importance of this problem in the development of quantum mechanics, even though it is becoming clear now that the Copenhagen preoccupation with measurement has been too restrictive and must be completed by a thorough consideration of preparation. It seems to me that Laudisa's judgment may stem from an overestimation of the value of Fine's above-mentioned theorem on the existence of a local deterministic hidden variables theory, the physical relevance of such a theory being questioned, for instance, by Garg and Mermin (1982).

In order to be able to deal also with individual preparations, I now turn once again to hidden variables theories. Stapp's point of view is illustrated most clearly[5] by considering the quadrivariate probability distribution (7). This distribution is not obtained from the relative frequencies of a joint measurement of the four observables, but is *theoretically* constructed using the conditional probabilities obtaining in the four separate EPR measurements and assuming the locality condition (5). In a quasi-objectivistic treatment it is thought to be possible to attribute to every individual preparation act each of the four conditional probability distributions $p_A(a_i|\lambda)$, $p_B(b_j|\lambda)$, $p_{A'}(a'_k|\lambda)$ and $p_{B'}(b'_l|\lambda)$ as probabilities realized if the relevant measurement is performed. These quantities are thought to be simultaneously attributable to the object, independently of which measurement is going to be performed. If this is possible, then each of the experimental bivariate probabilities (6) can be derived from (7), thus implying the Bell inequality.

As already discussed in Section 2, the reasoning leading to the probability distribution (7) not only presupposes locality but also a kind of objectivity called 'quasi-objectivity' above. Hence, Stapp's conclusion that only the locality assumption can be blamed for the derivability of the Bell inequality would only be justified if the assumption of quasi-objectivity were a necessary one. In this section I want to discuss the question whether it could be this objectivity assumption that is to be blamed. In order to do so, I will exploit the analogy between thermodynamics and quantum mechanics, these theories being considered as *phenomenological* theories describing a restricted class of phenomena within the domains of application of their underpinning 'microscopic' theories

[5] Note that Stapp eschews hidden variables and possibly may not be very happy with this illustration.

(viz classical statistical mechanics and the hidden variables theory, respectively). This analogy has been considered many times before (de Broglie, 1964; Bohm, 1953; Bohm and Vigier, 1954; Nelson, 1967; Nelson, 1985), and is employed here in the first place to demonstrate the logical possibility of hidden variables theories of a non-objectivistic character.

The basic idea is that quantum mechanical observables, like thermodynamic quantities, are not *instantaneous* properties of the object. Quantum measurements, like thermodynamic ones, take some time. Therefore they do not yield information on the object system valid for one particular instant of time, but yield *time-averaged* information. Hence, an individual measurement result, either quantum mechanical or thermodynamic, cannot be conditioned on the *microstate* [i.e. the hidden variable λ for quantum mechanics, or the phase space point $(\{q_n, p_n\})$ for thermodynamics]. In thermodynamics time averaging is replaced, under the ergodic hypothesis, by an ensemble averaging, the (canonical) ensemble being represented in the Gibbs theory by the canonical density function $Z^{-1}\,e^{-H(\{q_n,p_n\})/kT}$ on phase space. These so-called *macrostates* can be considered to represent the (restricted) class of ergodic trajectories (equilibrium states) described by thermodynamics. Indeed, thermodynamic quantities are functionals of the macrostate.[6]

Support for the idea that also quantum mechanics may describe only a restricted class of hidden variables states was found in de Muynck and van Stekelenborg (1988), in which it was demonstrated that in the Husimi representation of quantum mechanics by means of non-negative probability distribution functions on phase space a restriction to a 'canonical' set of distributions obtains, analogously to thermodynamics. In particular, it was demonstrated that the dispersion-free states $\rho(q, p) = \delta(q - q_0)\delta(p - p_0)$ are not 'canonical' in this sense. This implies that within the domain of quantum mechanics it does not make sense to consider the preparation of the object in a 'microstate' with a well-defined value of the hidden variable $\lambda = (q, p)$. A reasonable conjecture is that the 'canonical' states can be considered as 'macrostates', describing states of (local) equilibrium of the quantum system, analogous to the Gibbs states, averaging over the 'canonical' ensemble being, here also, equivalent to time averaging in the sense of an ergodic hypothesis.

If the analogy between quantum mechanics and thermodynamics is valid, then we should take into account the possibility that the measurements described by quantum mechanics do not probe the instantaneous hidden variable 'microstate' λ, but only refer to certain ergodic *trajectories* in hidden variables space Λ. This assumption would imply that for a hidden variables theory *describing quantum mechanical measurements* the conditional probabilities $p(a_i|\lambda)$, considered before, are irrelevant. They should be replaced by quantities relating the measurement results of a quantum mechanical observable with a 'macrostate', i.e.

[6] For instance, thermodynamic pressure is defined by $p = kT\partial/\partial V \log Z$. Note that the restriction to global equilibrium can be relaxed so as to have a thermodynamic description of states in which only *local* equilibrium is assumed.

$p(a_i|\overline{\lambda})$, in which the ergodic trajectory is indicated by $\overline{\lambda}$. Hence, the probability (3) should be replaced by

$$p(a_i) = \int p(a_i|\overline{\lambda})\ \rho(\overline{\lambda})\ \mathrm{d}\overline{\lambda}, \tag{9}$$

where the integration is over all possible 'macrostates', and, therefore, is a path integral rather than an integral over phase space. Contextuality due to measurement is representable, as before, by a possible dependence of the conditional probabilities on the measurement procedure.

Having used the thermodynamic analogy to find (9), it is necessary now to stress the difference between quantum mechanics and thermodynamics. Being a classical theory, in thermodynamics it is thought that the measurement procedure need not be considered explicitly. This can be implemented by making a sharp distinction between the preparation apparatus and the measurement apparatus, the definition of thermodynamic macrostates being governed only by the former one. This may be different in quantum mechanics. The essential role played by measurement in quantum mechanics makes it very probable that the 'macrostates' will be dependent also on the measurement arrangement, i.e. that quantum mechanical measurements only probe reality as far as equilibrium has been reached *in the presence of the measurement arrangement*. This has the important consequence that, even if we start with the same 'microstate' λ, the 'macrostates' will be different in different *measurement* arrangements:

$$\overline{\lambda}^A \neq \overline{\lambda}^{A'},\ A \neq A'. \tag{10}$$

Hence, (9) should be specified according to

$$p(a_i) = \int p(a_i|\overline{\lambda}^A)\ \rho(\overline{\lambda}^A)\ \mathrm{d}\overline{\lambda}^A. \tag{11}$$

This conjecture just tries to translate into the hidden variables theory Bohr's basic idea that quantum mechanical observables are well-defined only within the context of the measurement arrangement serving to measure the observable.

For EPR experiments we may suppose that locality now applies to the 'macrostates', thus yielding the following generalization of (6):

$$p_{AB}(a_i, b_j) = \int p_A(a_i|\overline{\lambda}^{AB})p_B(b_j|\overline{\lambda}^{AB})\ \rho(\overline{\lambda}^{AB})\ \mathrm{d}\overline{\lambda}^{AB}. \tag{12}$$

This blocks the theoretical construction of a quadrivariate probability analogous to (7), from which the Bell inequality was derived, because this construction is based on the assumption that in all EPR experiments the individual objects are prepared in the *same* state. It may be possible to assume that this is true for the 'microstates'. This, however, is irrelevant to measurements performed within the domain of quantum mechanics since such measurements do not probe the 'microstate' but the 'macrostate'. And there is no 'macrostate' common to all EPR experiments, because these experiments correspond to mutually exclusive experimental measurement arrangements. Only if it were possible to perform measurements probing the 'microstate', measurement results would be obtained satisfying the Bell inequality. Such measurements would have to be performed well within the relaxation time of 'subquantum' processes leading to the 'canonical' equilibrium states. This time is estimated by Bohm (1952, p. 179) 'for the sake of illustration' to be $10^{-13}/c$ second, c being the velocity of light. If this is correct, then such experiments are far outside the range of present-day experimental possibility. Quantum mechanical measurements are, hence, bound to violate the Bell inequality because they are too slow to be able to respond to the 'microstate' the object is prepared in.

In a sense this conjecture amounts to the same basic idea of a dependence of the preparation on the measurement arrangement as is involved in Lochak's proposal discussed in Section 2. There is an important difference, however. In the proposal presented here it is the *individual* state that is non-objective. With Lochak it is the *statistical* state $\rho(\lambda)$ that is thought to be dependent on the measurement arrangement, the hidden variable λ being treated objectively. The two approaches would be equivalent if it were possible to equate Lochak's contextual probability distribution $\rho_A(\lambda)$ with the probability distribution $\rho(\overline{\lambda})$ of (9). Actually, this is precisely what is done in Gibbs thermodynamics, where the macrostates $Z^{-1}\,e^{-H(\{q_n,p_n\})/kT}$ are treated as ordinary probability distributions on phase space, and the values of the thermodynamic quantities in such a macrostate are calculated as phase space averages using this distribution. It is at this time not clear whether the analogy between quantum mechanics and thermodynamics can be stretched so far as to allow the analogous procedure in quantum mechanics. In any case this procedure would be obscuring a physical understanding of the limits of the domain of validity, as much for quantum mechanics as it does for thermodynamics. In my opinion it is the Boltzmann-like approach considering the individual object, rather than the Gibbsian ensemble, that may teach us more about the nature of quantum mechanical measurements, and where to look in order to transcend the boundaries of the domain of application of quantum mechanics.

5. Conclusions

In this paper I have tried to demonstrate that the widespread belief that the quantum world is non-local, notwithstanding Laudisa's defence, is not based on firm ground. This, of course, does not imply that the quantum world must necessarily be local. It might very well be that the most elementary objects, or events, to be found in nature are not pointlike in the sense of the point particles of classical mechanics, and that this non-locality has some influence on certain measurements that are sensitive to this feature. This kind of non-locality, however, is completely different from the kind of non-locality involved in the Bell inequalities, where the non-locality spreads out over macroscopic distances. As discussed in Section 1 such a non-locality is utterly implausible in the face of all experimental evidence within the domain of quantum mechanics, including the Aspect experiments.

The central thesis of the present paper (Section 2) is that all conclusions to the effect that Bell non-locality is a reality stem from the consideration of models that are too restricted, even if it is taken into account that the restrictions to determinism and non-contextualism of the measurement procedures are relaxed by considering so-called stochastic contextualistic hidden variables theories. In Section 3, I discussed how the problem of the Bell inequality can be cast into a simple form employing the concept of a quadrivariate joint probability distribution. It is then easily seen that stochasticity and contextualism in the sense indicated above are not sufficient to thwart the derivation of the Bell inequality in local hidden variables theories.

Instead of concluding from this that it must be the assumption of Bell locality that is responsible (which is the basis of the widespread belief in Bell non-locality defended by Laudisa) I employed in Section 4 the analogy between quantum mechanics and thermodynamics to argue that hidden variables models reproducing the results of *quantum mechanical* measurements could still satisfy another property of non-objectivity, viz a non-objectivity in the initial individual preparation as far is this preparation is relevant to these particular measurements. It is demonstrated how this non-objectivity can block the derivation of the Bell inequality.

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