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Do absorber fluctuations reduce interference in neutron interferometry?

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A theory of joint nonideal measurement of incompatible observables is used to analyse absorber fluctuations in neutron interferometry. The theory is applied to two different fluctuation models. Also the influence of detector efficiency is discussed. Outcomes of the analysis are compared with conclusions based on the visibility of the interference pattern measured by one neutron detector only. It is demonstrated that the theory of joint measurement yields additional information on neutron interference experiments, thus making it a useful tool for analysing such experiments.

In neutron interference experiments nowadays both particle and wave aspects of quantum particles are studied in the same experiment. In experiments by Summhammer, Rauch and Tuppinger [1] this is done by inserting an absorber into one of the two interferometer beams (cf. fig. 1). In these experiments an incoming neutron into direction 1 or 2 can take either one of two possible paths through the interferometer. The detection probabilities of the neutron detectors D_A and D_B depend on the phase shift χ. If the absorber (transmission probability a) is absent the experiment is a pure interference measurement; insertion of the absorber makes it possible also to obtain some information on the path the neutron has taken through the interferometer.

By de Muynck and Martens [2] this experiment was analysed making use of the formalism of positive operator-valued measures (POVM). It was demonstrated to be interpretable as a joint nonideal measurement of quantum mechanical path and interference observables. In a two-dimensional representation defined by basis vectors $|L\rangle$ and $|R\rangle$ corresponding with the two possible neutron paths through the interferometer these observables are represented by the projection-valued measures

$$\{\hat{P}_+\hat{P}_-\},\quad \hat{P}_+=\frac{1}{2}\begin{pmatrix}1 & -\mathrm{i}\\ \mathrm{i} & 1\end{pmatrix},\quad \hat{P}_-=\hat{I}-\hat{P}_+ \quad \text{(path observable)}\,, \tag{1}$$

and

$$\{\hat{Q}_A,\hat{Q}_B\}\,,\quad \hat{Q}_A=\begin{pmatrix}\sin^2(\frac{1}{2}\chi) & \frac{1}{2}\sin(\chi)\\ \frac{1}{2}\sin(\chi) & \cos^2(\frac{1}{2}\chi)\end{pmatrix},$$

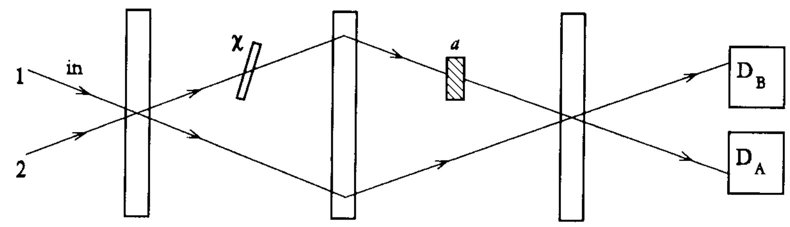

Fig. 1. Neutron interferometer for the joint measurement of interference and path. χ is the phase shift, a transmission probability of the absorber, D_A and D_B are neutron detectors.

$$\hat{Q}_B = \hat{I} - \hat{Q}_A \quad \text{(interference observable)}\,, \tag{2}$$

respectively. It was further shown that it is possible to find a bivariate POVM $\{\hat{R}_{mn}\}$ ($m=+, -$; $n=$A, B), the expectation values of which are determined by the detection probabilities found in the experiment, and such that its marginals $\sum_n \hat{R}_{mn}$ and $\sum_m \hat{R}_{mn}$ are nonideal measurements [3] of the path and interference observables, i.e.,

$$\sum_{n=\mathrm{A,\,B}} \hat{R}_{mn} = \sum_{k=+,-} \lambda_{mk} \hat{P}_k\,, \quad m=+,-\,, \tag{3}$$

$$\sum_{m=+,-} \hat{R}_{mn} = \sum_{l=\mathrm{A,\,B}} \mu_{nl} \hat{Q}_l\,, \quad n=\mathrm{A,\,B}\,, \tag{4}$$

$\hat{P}_k$ and $\hat{Q}_l$ being given by (1) and (2). The nonideality matrices (λ_{mk}) and (μ_{nl}) are stochastic matrices, reflecting the inaccuracy of the $\{\hat{R}_{mn}\}$ measurement as a joint measurement of the incompatible path and interference observables. We may choose

$$\hat{R}_{+\mathrm{A}} = \hat{M}_\mathrm{A}\,, \quad \hat{R}_{+\mathrm{B}} = \hat{M}_\mathrm{B}\,, \quad \hat{R}_{-\mathrm{A}} = \hat{R}_{-\mathrm{B}} = \tfrac{1}{2}\hat{M}_\mathrm{Z}\,, \tag{5}$$

in which $\hat{M}_\mathrm{A}$, $\hat{M}_\mathrm{B}$ and $\hat{M}_\mathrm{Z}$ are operators which, on taking expectation values in the initial state (which, in general, may be a superposition of incoming states 1 and 2), yield, respectively, the detection probabilities in the two outgoing beams A and B, and the absorption probability in the absorber (Z). These operators were derived in ref. [2]. They can be represented according to

$$\hat{M}_\mathrm{A} = \tfrac{1}{2}[\hat{P}_+ + a\hat{P}_- + \sqrt{a}\,(\hat{Q}_\mathrm{A} - \hat{Q}_\mathrm{B})]\,,$$
$$\hat{M}_\mathrm{B} = \tfrac{1}{2}[\hat{P}_+ + a\hat{P}_- - \sqrt{a}\,(\hat{Q}_\mathrm{A} - \hat{Q}_\mathrm{B})]\,,$$
$$\hat{M}_\mathrm{Z} = (1-a)\hat{P}_-\,. \tag{6}$$

From (3)–(6) the nonideality matrices (λ_{mk}) and (μ_{nl}) for the path and interference measurements are found as

$$(\lambda_{mk}) = \begin{pmatrix} 1 & a \\ 0 & 1-a \end{pmatrix},$$
$$(\mu_{nl}) = \frac{1}{2}\begin{pmatrix} 1+\sqrt{a} & 1-\sqrt{a} \\ 1-\sqrt{a} & 1+\sqrt{a} \end{pmatrix}, \tag{7}$$

respectively. Bohr's notion of complementarity is clearly expressed by these two matrices: if $a=0$ we have an accurate path measurement; if $a=1$ we have an accurate interference measurement; for intermediate values of *a both* measurements are inaccurate to a certain extent, the inaccuracies being quantified, for instance, by the deviations of the matrix determinants from their ideal values (equal to 1):

$$\epsilon_{(\lambda)} = a\,, \quad \epsilon_{(\mu)} = 1 - \sqrt{a}\,. \tag{8}$$

By eliminating a from (8) we obtain

$$\epsilon_{(\lambda)} = (1 - \epsilon_{(\mu)})^2\,. \tag{9}$$

Complementarity of interference and path measurements is demonstrated (cf. fig. 2) by the fact that (9) does not allow combinations near $\epsilon_{(\lambda)} = \epsilon_{(\mu)} = 0$.

In the foregoing it was assumed that the absorber is homogeneous having a stable transmission probability a. In a realistic experiment there will be spatial and temporal inhomogeneities, such that different fractions of the beam see different transmissions. If p_i is the fraction seeing transmission probability a_i, then we can take absorber fluctuations into account by taking, instead of (5),

$$\hat{R}_{+\mathrm{A}} = \sum_i p_i \hat{M}_\mathrm{A}(a_i) = \hat{\bar{M}}_\mathrm{A}\,,$$
$$\hat{R}_{+B} = \sum_i p_i \hat{M}_\mathrm{B}(a_i) = \hat{\bar{M}}_\mathrm{B}\,,$$
$$\hat{R}_{-\mathrm{A}} = \hat{R}_{-\mathrm{B}} = \tfrac{1}{2}\sum_i p_i \hat{M}_\mathrm{Z}(a_i) = \tfrac{1}{2}\hat{\bar{M}}_\mathrm{Z}\,. \tag{10}$$

The operators $\hat{\bar{M}}_\mathrm{A}$, $\hat{\bar{M}}_\mathrm{B}$, and $\hat{\bar{M}}_\mathrm{Z}$ can be obtained from (6) by replacing a and $\sqrt{a}$ by their averages

$$\bar{a} = \sum_i p_i a_i\,, \quad \overline{\sqrt{a}} = \sum_i p_i \sqrt{a_i}\,. \tag{11}$$

The nonideality matrices $(\bar{\lambda}_{mk})$ and $(\bar{\mu}_{nl})$ for the case of the fluctuating absorber are found analogously to (7). They, also, turn out to be obtainable from (7) by taking the averaged quantities (11) instead of a and $\sqrt{a}$, respectively. This implies that the inaccuracy measures (8) are changed into

$$\epsilon_{(\bar{\lambda})} = \bar{a}\,, \quad \epsilon_{(\bar{\mu})} = 1 - \overline{\sqrt{a}}\,. \tag{12}$$

According to the theory of joint measurement absorber fluctuations reduce the quality of the experiment by changing the inaccuracy measures $\epsilon_{(\lambda)}$ and $\epsilon_{(\mu)}$ so as to move away from the point $\epsilon_{(\lambda)} = \epsilon_{(\mu)} = 0$ (cf. fig.2). This can be seen as follows. From the Schwartz inequality it follows that

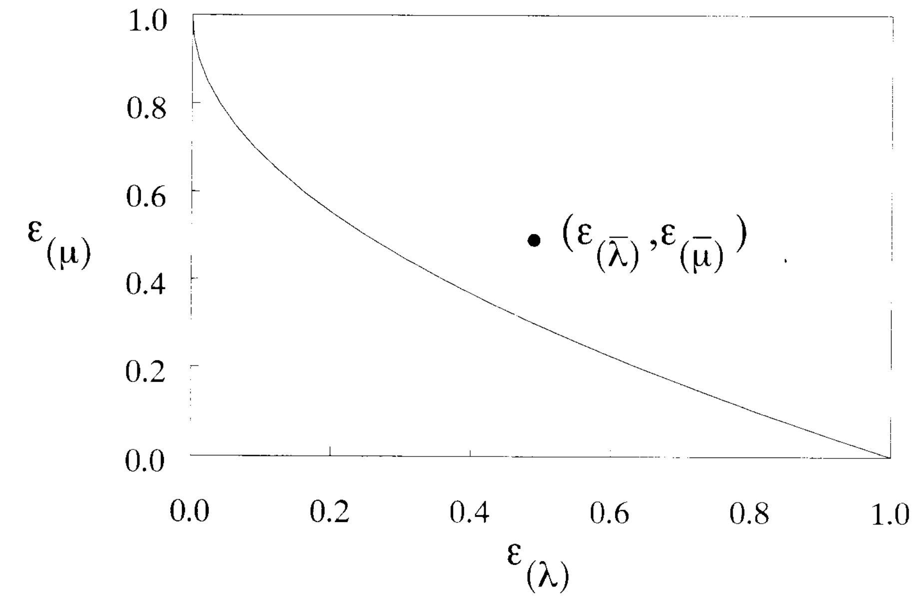

Fig. 2. Complementarity of path and interference measurements (without fluctuations (——); $(\epsilon_{(\bar\lambda)}, \epsilon_{(\bar\mu)})$ is a generic point for the fluctuating case).

$$\left(\sum_i p_i \sum_j p_j a_j\right)^{1/2} \geqslant \sum_i p_i^{1/2}(p_i a_i)^{1/2} .$$

With (11) and (12) this immediately implies

$$\epsilon_{(\bar\lambda)} \geqslant (1-\epsilon_{(\bar\mu)})^2 . \tag{13}$$

It must be noted that this conclusion is independent of the question whether the transmission probability a is changed by the fluctuations, or not.

In the following we will compare two different models that may be satisfied by the absorber fluctuations. In the first model it is assumed that the mean transmission probability is not changed by the fluctuations. Hence,

$$\bar a = a , \tag{14}$$

implying

$$\epsilon_{(\bar\lambda)} = \epsilon_{(\lambda)} . \tag{15}$$

So, in this model the accuracy of the path measurement is not influenced by the fluctuations. It immediately follows from (13) that

$$\epsilon_{(\bar\mu)} \geqslant \epsilon_{(\mu)} , \tag{16}$$

equality holding only in the homogeneous case. Hence, under condition (14) the inaccuracy of the interference measurement is increased by the absorber fluctuations. The reduction of interference can also be seen from the intensity $I_{\hat{\bar M}_A}$ measured by detector A if the incoming beam is $|R\rangle$ (this is the input state applied in most neutron interference experiments):

$$I_{\hat{\bar M}_A} \sim \langle R|\hat{\bar M}_A|R\rangle = \tfrac14[1+\bar a+2\overline{\sqrt{a}}\cos(\chi)] . \tag{17}$$

Since, because of (16), $\overline{\sqrt{a}} \leqslant \sqrt{a}$, (17) shows a decreased interference amplitude as compared with the homogeneous case. Indeed, comparing the visibility $V_{\hat{\bar M}_A}$ of this interference pattern with the one obtained from a nonfluctuating absorber, we obtain, because of (14),

$$V_{\hat{\bar M}_A} = \frac{I_{\hat{\bar M}_A\max} - I_{\hat{\bar M}_A\min}}{I_{\hat{\bar M}_A\max} + I_{\hat{\bar M}_A\min}} = \frac{2\overline{\sqrt{a}}}{1+\bar a} \leqslant V_{\hat M_A} = \frac{2\sqrt{a}}{1+a} , \tag{18}$$

the unbarred quantities referring to the nonfluctuating absorber. So, in concordance with the decrease of the interference amplitude the visibility is diminished by the fluctuations. It is also worth noting that the visibility has its maximal value $V=1$ if $a=1$, thus expressing the disturbance of the interference by the mere presence of the absorber. Evidently, in accordance with Bohr's complementarity principle, the interference is reduced, independently of any absorber fluctuation, by the presence of that part of the measurement arrangement that is intended to measure the path observable.

It will be important to discuss the reduction of interference from yet another point of view. In the formalism describing the joint nonideal measurement of interference and path, as realised by eqs. (3)–(6) for the nonfluctuating case, or by (3)–(5) and (10) for the fluctuating absorber, the outcome probabilities of the nonideal interference experiment are given by the expectation values of the marginals (4) (and analogously by (3) for the path observable). From (4) and (6) the relevant quantity determining the (nonideal) interference pattern is found to be the probability distribution $\mathrm{Tr}\,\hat{\rho}(\hat{M}_{\mathrm{A}}+\frac{1}{2}\hat{M}_{\mathrm{Z}})$, or its averaged version in case of absorber fluctuations. For $\hat{\rho}=|\mathrm{R}\rangle\langle\mathrm{R}|$ we get for this quantity

$$\langle\mathrm{R}|\hat{M}_{\mathrm{A}}+\tfrac{1}{2}\hat{M}_{\mathrm{Z}}|\mathrm{R}\rangle=\tfrac{1}{2}[1+\sqrt{a}\cos(\chi)]$$

$$\text{(no fluctuations)}\,, \tag{19}$$

$$\langle\mathrm{R}|\hat{\hat{M}}_{\mathrm{A}}+\tfrac{1}{2}\hat{\hat{M}}_{\mathrm{Z}}|\mathrm{R}\rangle=\tfrac{1}{2}[1+\overline{\sqrt{a}}\cos(\chi)]$$

$$\text{(fluctuating absorber)}\,. \tag{20}$$

Comparing these quantities with (17) and its nonfluctuating analogue, we see that they essentially contain the same information about the interference, the interference amplitudes being the same in the physically identical cases. For the visibilities of (19) and (20) we obtain, once again using (14),

$$V_{\hat{\hat{M}}_{\mathrm{A}}+\hat{\hat{M}}_{\mathrm{Z}}/2}=\overline{\sqrt{a}}\leqslant V_{\hat{M}_{\mathrm{A}}+\hat{M}_{\mathrm{Z}}/2}=\sqrt{a}\,, \tag{21}$$

which is qualitatively not very different from (18). In particular also this quantity has $V=1$ if $a=1$. For this reason it would seem not to make any difference whether the analysis is based on the output probability of detector A (cf. (17)) or on the probability distribution described by $\hat{\hat{M}}_{\mathrm{A}}+\frac{1}{2}\hat{\hat{M}}_{\mathrm{Z}}$ (cf. (20)). The direct experimental availability may probably have encouraged the use of the first quantity.

We will now discuss an argument in favour of the second one, viz. (20). To this end we take into account the inefficiency of the neutron detectors, that can be considered as a source of nonideality of the measurement in excess of the ones discussed before. If $\eta<1$ is the efficiency of the detectors A and B (for simplicity being taken equal), then we have to multiply $\mathrm{Tr}\,\hat{\rho}\hat{M}_{\mathrm{A}}$ and $\mathrm{Tr}\,\hat{\rho}\hat{M}_{\mathrm{B}}$ by η in order to obtain the detection probabilities of the detectors. Hence, the POVM is changed into $\{\hat{\hat{M}}'_{\mathrm{A}},\hat{\hat{M}}'_{\mathrm{B}},\hat{\hat{M}}'_{\mathrm{Z}}\}$, in which $\hat{\hat{M}}'_{\mathrm{A}}=\eta\hat{\hat{M}}_{\mathrm{A}}$, $\hat{\hat{M}}'_{\mathrm{B}}=\eta\hat{\hat{M}}_{\mathrm{B}}$ describe the reduced detection probabilities of the detectors and $\hat{\hat{M}}'_{\mathrm{Z}}$ now not only refers to the absorption in the absorber, but also to the losses due to the inefficiency of the detectors. Since $\hat{\hat{M}}'_{\mathrm{Z}}=\hat{I}-\hat{\hat{M}}'_{\mathrm{A}}-\hat{\hat{M}}'_{\mathrm{B}}$, its expectation values can be found experimentally from the detection rates of *both* detectors A and B.

It is to be expected that losses due to detector inefficiency make the interference measurement less accurate. Indeed, if we apply the theory of joint measurement to the POVM $\{\hat{\hat{M}}'_{\mathrm{A}},\hat{\hat{M}}'_{\mathrm{B}},\hat{\hat{M}}'_{\mathrm{Z}}\}$ instead (6), we obtain the following results. The measurement remains a joint nonideal measurement of path and interference, the nonideality matrices (7) changing into

$$(\bar{\lambda}'_{mk})=\begin{pmatrix}\eta & \eta\bar{a}\\ 1-\eta & 1-\eta\bar{a}\end{pmatrix},$$

$$(\bar{\mu}'_{nl})=\frac{1}{2}\begin{pmatrix}1+\eta\overline{\sqrt{a}} & 1-\eta\overline{\sqrt{a}}\\ 1-\eta\overline{\sqrt{a}} & 1+\eta\overline{\sqrt{a}}\end{pmatrix}, \tag{22}$$

yielding as inaccuracy measures (cf. (12))

$$\epsilon_{(\bar{\lambda}')}=1-\eta(1-\bar{a})\,,\quad \epsilon_{(\bar{\mu}')}=1-\eta\overline{\sqrt{a}}\,. \tag{23}$$

From (23) we see that the quality of both path and interference measurements is diminished by detector inefficiency. Reduction of interference can also be seen by taking into account the effect of detector inefficiency in (20). Then we get

$$\langle\mathrm{R}|\hat{\hat{M}}'_{\mathrm{A}}+\tfrac{1}{2}\hat{\hat{M}}'_{\mathrm{Z}}|\mathrm{R}\rangle=\tfrac{1}{2}+\tfrac{1}{2}\eta\overline{\sqrt{a}}\cos(\chi)\,, \tag{24}$$

reducing the visibility (21) to

$$V_{\hat{\hat{M}}'_{\mathrm{A}}+\hat{\hat{M}}'_{\mathrm{Z}}/2}=\eta\overline{\sqrt{a}}\,. \tag{25}$$

On the other hand detector inefficiency does not have any influence on the visibility of the output of detector A because η enters as a multiplicative constant in this output, bias term $\frac{1}{4}(1+\bar{a})$ and interference amplitude $\sqrt{\bar{a}}$ in (17) being multiplied by the same factor η. Hence, η drops out of $V_{\hat{M}'_A}$, thus yielding (cf. (18))

$$V_{\hat{M}'_A} = V_{\hat{M}_A} . \tag{26}$$

Although from an experimental point of view the η-independence of the quantity (26) may be advantageous, in view of (23) this very property seems to render this quantity less useful as a measure of the quality of the experiment.

As seen from (24) for the quantity $\langle R|\hat{\bar{M}}'_A + \frac{1}{2}\hat{\bar{M}}'_Z|R\rangle$ the bias term is independent of η, thus leading to the η-dependent visibility (25). In our opinion it is the latter quantity that, better than (17), indicates the quality of the interference measurement. The quantity (24) does not only take into account information based on the detection rate of detector A but also depends on the loss rate described by $\hat{\bar{M}}'_Z$. If there are no other losses than those in the absorber Z and the detectors A and B, it is not necessary to measure these losses directly (e.g., by means of direct detection of the absorption in Z) in order to evaluate (24). Instead, since

$$\mathrm{Tr}\,\hat{\rho}(\hat{\bar{M}}'_A + \tfrac{1}{2}\hat{\bar{M}}'_Z) = \tfrac{1}{2} - \tfrac{1}{2}(\mathrm{Tr}\,\hat{\rho}\hat{\bar{M}}'_A - \mathrm{Tr}\,\hat{\rho}\hat{\bar{M}}'_B) , \tag{27}$$

it suffices to measure the detection rates of *both* detectors A and B. By taking into account the detection rate of one detector only, the quantity (17) does seem to ignore part of the information that is present in the experiment. The theory of joint measurement of incompatible observables is indicating a way to make a more comprehensive use of this information by taking (27) as the basis of an analysis of the interference experiment rather than (17).

It is straightforward to generalize the joint measurement treatment to *different* detector efficiencies η_A and η_B. To this end it is appropriate to take $(\eta_A\eta_B)^{1/2}\hat{\bar{M}}_A$ and $(\eta_A\eta_B)^{1/2}\hat{\bar{M}}_B$ as the components $\hat{R}_{+A}$ and $\hat{R}_{+B}$, respectively, of the bivariate POVM $\{\hat{R}_{mn}\}$ rather than the operators $\eta_A\hat{\bar{M}}_A$ and $\eta_B\hat{\bar{M}}_B$ representing the detection probabilities (cf. (10)). For the former choice the marginals (3) and (4) once again describe nonideal measurements of path and interference. The nonideality matrices are found to be obtainable from (22) by substituting $(\eta_A\eta_B)^{1/2}$ for η.

As a second model we consider a model discussed by Namiki and Pascazio [4]. Contrary to (14) in their model the transmission probability is changed by the absorber fluctuations. Their analysis is based on the visibility of the interference pattern measured directly by detector A, analogous to (17). Their conclusion is that, since the visibility of this interference pattern is decreased by the fluctuations, the interference measurement becomes a nonideal one. This conclusion appears to be in accord with the results obtained above. However, in view of our reservations with respect to the feasibility of the quantity (26) as a measure of the quality of the interference experiment it seems appropriate to apply the theory of joint measurement also to this model.

In order to calculate in the Namiki–Pascazio model the effect of the absorber fluctuations on the POVM $\{\hat{R}_{mn}\}$ we have to perform, analogously to the procedure of Namiki and Pascazio [4], in (6) the substitution $\sqrt{a}\to\sqrt{a}\,(1+\Delta)$. After averaging over the fluctuations (such that $\bar{\Delta}=0$ and $\overline{\Delta^2}=\epsilon$) this leads to the following POVM,

$$\hat{\bar{M}}''_A = \tfrac{1}{2}[\hat{P}_+ + a(1+\epsilon)\hat{P}_- + \sqrt{a}\,(\hat{Q}_A - \hat{Q}_B)] ,$$
$$\hat{\bar{M}}''_B = \tfrac{1}{2}[\hat{P}_+ + a(1+\epsilon)\hat{P}_- - \sqrt{a}\,(\hat{Q}_A - \hat{Q}_B)] ,$$
$$\hat{\bar{M}}''_Z = [1 - a(1+\epsilon)]\hat{P}_- . \tag{28}$$

Analogously to (10) we find that this experiment can also be interpreted as a joint nonideal measurement of path and interference in the sense of eqs. (3) and (4), the nonideality matrices (7) being changed into

$$(\bar{\lambda}''_{mk}) = \begin{pmatrix} 1 & a(1+\epsilon) \\ 0 & 1-a(1+\epsilon) \end{pmatrix}, \quad (\bar{\mu}''_{nl}) = (\mu_{nl}) . \tag{29}$$

The inaccuracy measures (8) become

$$\epsilon_{(\bar{\lambda}'')} = a(1+\epsilon) > \epsilon_{(\lambda)} ,$$
$$\epsilon_{(\bar{\mu}'')} = 1 - \sqrt{a} = \epsilon_{(\mu)} . \tag{30}$$

Hence, also in this model (13) is satisfied, thus demonstrating a disturbing influence of the absorber fluctuations on the measurement. However, now the fluctuations turn out only to affect the accuracy of the path measurement, the interference measurement being unaffected.

The independence of interference from the absorber fluctuations in the Namiki-Pascazio model is also seen from an analysis of the quantity $\mathrm{Tr}\,\hat{\rho}(\hat{\hat{M}}''_{\mathrm{A}}+\frac{1}{2}\hat{\hat{M}}''_{\mathrm{Z}})$, analogous to (20). For $\hat{\rho}=|\mathrm{R}\rangle\langle\mathrm{R}|$ we obtain

$$\mathrm{Tr}\,\hat{\rho}(\hat{\hat{M}}''_{\mathrm{A}}+\tfrac{1}{2}\hat{\hat{M}}''_{\mathrm{Z}})=\tfrac{1}{2}[1+\sqrt{a}\cos(\chi)]\,, \qquad (31)$$

which, contrary to the quantity $\mathrm{Tr}\,\hat{\rho}\hat{\hat{M}}''_{\mathrm{A}}$ considered by Namiki and Pascazio, is independent of ϵ. Comparing (31) with

$$\langle\mathrm{R}|\hat{\hat{M}}''_{\mathrm{A}}|\mathrm{R}\rangle=\tfrac{1}{2}[1+a(1+\epsilon)+\sqrt{a}\cos(\chi)]\,, \qquad (32)$$

which corresponds with eqs. (6)–(8) of ref. [4], we see that the inference amplitudes of the two interference patterns (31) and (32) are equal, as was the case in the first model discussed in this Letter. Contrary to the result obtained there, in the Namiki–Pascazio model these amplitudes are independent of the fluctuations. The decreased visibility found by Namiki and Pascazio is due to an excess χ-independent bias term in (32), stemming from a fluctuation induced increase, obtained in their model, of the average transmission probability of the absorber. Evidently the term $\frac{1}{2}\langle\mathrm{R}|\hat{\hat{M}}''_{\mathrm{Z}}|\mathrm{R}\rangle$ in (31) precisely compensates for this excess bias term.

Summarizing, it seems to us that the conclusion by Namikin and Pascazio that absorber fluctuations reduce neutron interference is not a cogent one. If the information of both detectors A and B is taken into account as in (31), their model turns out not to yield a reduction of interference by the absorber fluctuations, as also suggested by the fluctuation independence of the interference amplitude of $\langle\mathrm{R}|\hat{\hat{M}}''_{\mathrm{A}}|\mathrm{R}\rangle$. This latter result seems to follow from an assumption, made in their model, of homogeneity of the fluctuations (eq. (25) of ref. [4]). This assumption is sufficient to leave the average amplitude of the wave function transmitted by the absorber unchanged as compared with the nonfluctuating case. In actual experiments this homogeneity assumption may not be satisfied, thus yielding a reduction of the interference as following from (14). The analogous conclusion of Namiki and Pascazio, in our opinion, can follow from their model only if part of the information present in the experiment is ignored by only looking at the visibility of the output of detector A. An analysis on the basis of the theory of joint measurement automatically takes into account all information, and seems to be more suitable to assess the relative merits of different assumptions made. It is straightforward to take into account detector inefficiency also in this model.

As follows from the foregoing discussion both absorber fluctuations and detector inefficiency may have a deteriorating effect on the measured interference pattern. This does not imply, however, that any quantum mechanical information about the incoming state $\hat{\rho}$ as regards interference or path is irretrievably lost. As a matter of fact, if the outcome probabilities $\mathrm{Tr}\,\hat{\rho}\hat{\hat{M}}'_{\mathrm{A}}$ and $\mathrm{Tr}\,\hat{\rho}\hat{\hat{M}}'_{\mathrm{B}}$ of *both* detectors A and B are known (and, hence, also $\mathrm{Tr}\,\hat{\rho}\hat{\hat{M}}'_{\mathrm{Z}}$), the theory of joint measurement allows one to calculate the ideal probabilities $\mathrm{Tr}\,\hat{\rho}\hat{P}_m$ $(m=+,\ -)$ and $\mathrm{Tr}\,\hat{\rho}\hat{Q}_n$ $(n=\mathrm{A},\ \mathrm{B})$ for the observables (1) and (2), respectively, by inverting relations (3) and (4). This is possible because the matrices (22) have inverses unless $\eta=0$ or $\bar{a}$ has the value 0 or 1 (in the latter case only the path observable cannot be determined exactly). Hence, even in the case of a fluctuating absorber and inefficient detectors the experiment can yield correct statistical information on the interference observable. Evidently this information is not wiped out either by fluctuations in the absorber or by the inefficiency of the detectors. It only gets stored in the experimental outcomes in a somewhat more involved way.

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