ON THE INTERPRETATION OF QUANTUM MECHANICS AND RELATIVITY THEORY

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ABSTRACT

Realist and empiricist interpretations of quantum mechanics and relativity theory are compared. Methodological and physical arguments are discussed favoring an empiricist interpretation of quantum mechanics. Analogous arguments are found for an empiricist interpretation of relativity theory.

1. Introduction

In most textbooks on quantum mechanics and relativity theory a *realist* interpretation of the theory is entertained, in which the theory is thought to refer to (microscopic) reality more or less in the same way as this is thought to hold for classical mechanics. Thus, electrons are thought of as wave packets flying around in space; relativistic events $(\mathbf{x}, t)$ are considered as evidence of the presence of a particle or a photon at that particular point in space-time. Yet, both in the original developments of quantum mechanics and relativity theory strong empiricist elements can be found, pointing toward an interpretation of the theories as a description of *observable phenomena.* This is especially clear in quantum mechanics where physical properties are indicated as "observables", to be distinguished sharply from any property to be attributed to a "reality behind the phenomena". In quantum mechanics also the role of the measuring instrument is stressed, suggesting an *empiricist* interpretation of the measured value of an observable, viz., as the result of a pointer reading of a (macroscopic) pointer of the measuring instrument. In textbooks, however, such a pointer reading is generally equated with the determination of a property of the microscopic object itself, thus attributing to the observable a *realist* significance.

It is the purpose of the present contribution to put some question marks behind this identification of pointer positions and properties of the object, and behind the tendency that can be observed in both quantum mechanics and relativity theory toward a realist interpretation of the mathematical formalism. In this paper I want to consider in detail a number of measurement procedures in quantum mechanics and in relativity theory, strongly suggesting that a realist interpretation is not generally possible, and that an empiricist interpretation may be the more natural one.

2. Realist and Empiricist Interpretations of Physical Theories

In order to be to be able to define the notions of realist and empiricist interpretations we must distinguish between two different levels of reality, viz., the reality of the phenomena, involving the macroscopically observable parts of measuring instruments and preparing apparata, and the reality of the (microscopic) object. In an empiricist interpretation the theory is thought to describe just (cor)relations between observable phenomena corresponding with preparation and detection events, i.e., between input and output of a measuring process. Although the (microscopic) object certainly need not be considered as nonexisting (as is sometimes implied by positivist thinkers), and may be thought of as conveying information from the preparing apparatus to the measuring instrument by means of a physical process, the object itself is not represented in the theory (cf. fig. 1). Thus, in quantum mechanics the two mathematical quantities determining the detection probabilities obtained in a measurement, viz., the density operator ρ and the observable A having values a_m, are thought of as merely labels of preparing and measuring instruments, respectively, the values a_m labeling the possible pointer positions. Analogously, in an empiricist interpretation of relativity theory geometrical objects (tensors) are thought to describe just readings of measuring instruments, i.e., of measuring rods and clocks. A world line $x^\alpha(\tau)$ just represents the result of a continuous observation, i.e., the readings of clocks and measuring rods used in a detection process observing the path a relativistic object takes through space-time.

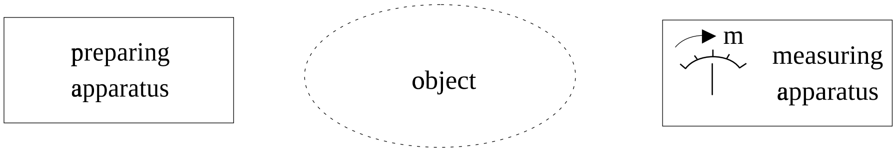

Figure 1: Empiricist reality.

In a realist interpretation the theory is thought to describe the (microscopic) object itself*. Although in this latter interpretation the same preparing and measuring apparata are used as in the empiricist one, now these apparata are not thought to be represented in the theory (cf. fig. 2), unless, of course, these apparata are themselves

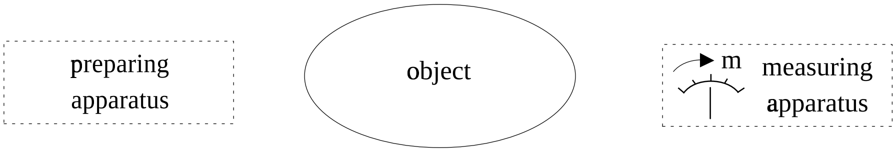

Figure 2: Realist reality.

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[*] Note that the nomenclature is rather unfortunate because in both the realist and the empiricist interpretation of quantum mechanics the theory is thought to describe a part of reality, namely on the one hand the reality of the microscopic object and on the other hand the macroscopic reality of the measurement arrangement. Although these parts are different they may be considered equally real. In this sense an empiricist interpretation is as "realist" as a realist one. Nevertheless I will stick to this nomenclature because it is generally used.

taken as objects of the theoretical description. Theoretical concepts are thought to refer to properties of the (microscopic) object. Thus, in quantum mechanics the state function is interpreted as a description of the state of the microscopic object, and values a_m of observable A are possible properties of this object. In a realist interpretation of relativity theory tensors are thought to describe properties of physical objects; the world line $x^\alpha(\tau)$ is interpreted as the physical path of a particle or photon through space-time. In the realist interpretation the role of the measuring instrument is minimized. It is not thought to have a disturbing influence on the information that is obtained: the measurement is thought to reveal reality "as it is". At most the measurement arrangement serves to yield a context with respect to which an observable can take on real values (contextualistic realism): the measurement reveals reality "as it is in the context of the measurement arrangement".

3. Axiomatic Treatment of Measurement in Quantum Mechanics and Relativity Theory

Both in textbook quantum mechanics and in textbook relativity theory measurement is described axiomatically. Thus, in the Dirac-von Neumann axiomatization of quantum mechanics an observable corresponds with a selfadjoint operator, $A = \sum_m a_m E_m$, its spectral representation $\{E_m\}$ defining the probability distribution, found on measuring the observable, according to

$$p_m = Tr\rho E_m. \tag{1}$$

It is remarkable that in this definition no trace is found of the interaction between object and measuring instrument thought to be so essential for an understanding of fundamental characteristics of quantum mechanics like complementarity and the uncertainty principle. This very fact seems to be at the basis of the tendency, observed above, to interpret the quantum mechanical observable in a realist way. Taking into account that within the domain of quantum mechanics interaction with a measuring instrument is necessary in order to be able to obtain information on the object, it would seem appropriate to consider the interaction with the object of any measuring instrument intended to measure a certain observable, and see whether (1) is satisfied. We shall call (1) a measurement postulate because it actually *prescribes* which measurement results a measuring instrument should yield in order to be functioning as a measuring instrument for the quantum mechanical observable A.

Measurement schemes satisfying the quantum mechanical measurement postulate are known to exist theoretically. In a so-called measurement of the first kind the measurement interaction transforms the initial state $\Psi_i = \psi\theta^a_0$ of object and measuring apparatus (θ^a_0 the initial state of the measuring instrument) into the final state $\Psi_f = \sum_m c_m \phi_m \theta^a_m$, in which ϕ_m is the eigenfunction of A corresponding with eigenvalue a_m , $c_m = \langle\phi_m|\psi\rangle$, and θ^a_m is the $m-$th pointer state. From Ψ_f the probability of a final pointer position θ^a_m can be calculated by applying (1) to the final state of the measuring apparatus, yielding $|c_m|^2$, in agreement with the probability axiomatically prescribed through (1) by quantum mechanics as applied to the object alone.

For relativity theory the situation as regards measurement is completely analogous. Also here we have a measurement postulate prescribing what are the correct relativistic measurement procedures for length and time, viz., the requirement

$$\nabla g = 0 \tag{2}$$

that the covariant derivative of the metric tensor vanish. This requirement warrants that measuring instruments do not change under parallel transport along geodesics. It is also easily seen that it governs Lorentz contraction and time dilation if the parallel transport is along a nongeodesic world line. Hence (2) is a prescription how to compare lengths of measuring rods and rates of clocks in different frames of reference, thus, actually, defining which measurement results a relativistic measuring instrument should produce in order to be functioning as a relativistic clock or measuring rod.

Also measuring instruments satisfying the relativistic measurement postulate do exist theoretically. Thus, a Born rigid object, each element of which satisfying the Lorentz contraction, can be considered as an ideal measuring rod in relativity theory; a geodesic clock is synchronized in agreement with (2).

4. Generalized Observables and Interpretations of Quantum Mechanics

Realistic measurement procedures within the domain of quantum mechanics seldom even approximately resemble a measurement of the first kind. A conventional photon counter, determining the number of photons present in an electromagnetic field is a widely used counterexample. Far from preserving the photon number as would be required by a measurement of the first kind, in such a photon counter all photons are annihilated if its efficiency ξ equals 1. If $\xi < 1$ there is a nonzero probability that a fraction of the photons remains undetected[†]. It appears that as the photon counter better serves its purpose of registering precisely the number of photons present, it must be less similar to a measurement of the first kind.

According to the Dirac-von Neumann axiomatization of quantum mechanics a photon counter must be described by a selfadjoint operator, viz., the number operator $N = \sum_0^\infty n N_n$, in which $N_n = |n\rangle\langle n|$ is the projection operator on the state of n photons. According to (1) the probability of finding n photons must be given by $p_n = Tr\rho N_n$. By Loudon[1] an expression is given for the detection probability of a photon counter having $\xi < 1$, which can be written down according to

$$p_m = Tr\rho M_m, \tag{3}$$

$$M_m = \sum_{n=m}^{\infty} \binom{n}{m} \xi^m (1-\xi)^{n-m} N_n. \tag{4}$$

[†] In order to avoid too sophisticated language I employ here the realist idiom of objects being in states. By considering a measurement of the first kind as a preparing process for the final state of the object it would, however, be equally well possible to employ the empiricist terminology of states as labels of preparing processes.

For $\xi = 1$ this can be seen to reduce to $M_m = N_m$. Hence, in this limit the measurement satisfies the Dirac-von Neumann axiomatization even though it is not a measurement of the first kind. The set of operators $\{M_m\}$ satisfies

$$M_m \geq O, \sum_{m=0}^{\infty} = I, \tag{5}$$

which characterizes it as a set of operators defining a positive operator-valued measure (POVM)[2,3]. If the operators M_m are projection operators the POVM is called a projection-valued measure (PVM). Only in this latter case it is possible to associate a selfadjoint operator with the measurement. In general, however, a quantum mechanical measurement is described by a POVM rather than by a selfadjoint operator. A POVM is said to represent a generalized observable.

In Ref. 4 the relation (4) was generalized to express the notion of an observable $\{M_m\}$ describing a *nonideal* measurement of observable $\{N_n\}$. This notion of nonideality induces a partial ordering in the set of generalized observables, maximal Dirac-von Neumann observables being also maximal in the sense of the partial ordering. Hence, such observables yield information the quality of which is maximal in a certain sense. This, however, does not imply that from an operational point of view the subset of observables represented by PVM's would take any special position. As a matter of fact, it was also demonstrated that maximal generalized observables exist, not corresponding to a PVM, that are superior to any Dirac-von Neumann observable with respect to the quantity of information provided by a measurement. It also must be noted that, although relations like (4) express a certain ordering as to the quality of photon counting measurements performed with different efficiencies, they often can be inverted. Thus, from (4) we obtain the inverted relation

$$N_n = \sum_{m=n}^{\infty} \binom{m}{n} (-1)^{m-n} \xi^{-m} (1-\xi)^{m-n} M_m, \tag{6}$$

enabling the experimenter to obtain information on the maximal observable $\{N_n\}$ by performing a measurement of the nonmaximal observable $\{M_m\}$. It should be noted that nonideality relations are not always invertible. When the inverse relation does not exist, then the information provided by the two measurements is inequivalent.

In a realist interpretation of quantum mechanics the probability $p_n = Tr\rho N_n$ is interpreted as the probability that n photons are present in the electromagnetic field. A natural interpretation of the probabilities (3) could be given by the assumption that in an inefficient detection process only a fraction of the photons present is registered. This actually is in agreement with the way (4) is derived, each photon being assumed to have a probability ξ to be registered by the counter. Note, however, that this interpretation essentially is an empiricist one, the probabilities now referring not to the object (the e.m. field) but to the measuring instrument.

Since we do not have any reason to attribute a more realistic significance to Dirac-von Neumann observables than to generalized observables, a discontinuity in the interpretation of observables as suggested by the photon counter example does not seem to be acceptable in general. This leaves us with two possibilities: i) either interpret generalized observables realistically; for the inefficient photon counter this would imply the notion of an unsharp or fuzzy reality[5,6], the measurement result m corresponding with an unsharp number of photons present in the e.m. field; ii) or, interpret both generalized and Dirac-von Neumann observables in the empiricist sense; then both the measurement results of the efficient and the inefficient photon counter are to be interpreted as observations of photon counting events (clicks in a photon counter), *not* to be identified with microscopic objects.

Although the first choice cannot be falsified and may be a possible one in principle, it seems to me that the second choice is the preferable one. As a matter of fact, it is the quantum mechanical interaction with a measuring instrument which leads to the POVM (4). In a realist interpretation of this observable the probabilistic effect of this interaction is attributed to the object as an unsharp property. Even if it is not in itself contradictory to interpret such an unsharp property in a contextualistic sense as a real property of the object, it seems much more natural to interpret the unsharpness as a property of the measuring process.

Also from a methodological point of view the empiricist interpretation seems preferable, at least if it is a good methodological principle to require that a theoretical term in a physical theory should be attributed at most one single well-defined meaning. In a realist interpretation of quantum mechanics any observable must necessarily have two different meanings, viz., both referring to a property of the object and to a property of the measuring instrument (its pointer position). This dual role of the quantum mechanical observable in a realist interpretation cannot be avoided because it is impossible to obtain quantum mechanical information without using a measuring instrument. An empiricist interpretation of quantum mechanics does not have this drawback because in this interpretation the theory is not thought to describe the microscopic object.

5. Nonideal Measurement and the Interpretation of Relativity Theory

It is seldom discussed in textbooks of relativity theory that a similarity may exist between the interpretational problems of quantum mechanics and relativity theory. Results of relativistic length and time measurements are generally interpreted in a realist sense as properties to be attributed to the object, even though it is realized that there is no *absolute* reality in the sense of absolute time or absolute length. Length and time are specified with respect to a frame of reference, the latter often being equated with an observer sitting in this frame of reference. Analogous to axiomatic quantum mechanics, also in axiomatic relativity theory the measurement performed by the observer is not treated as a physical process. In general also here meter readings are not distinguished from properties of the object itself. Of course, reality looks different in different frames of reference. This, however, is seldom attributed to the different ways measurements are performed. It is accepted that absolute reality within the domain of relativity theory must be replaced by the notion of *relative reality*, each observer (reference frame) having his own reality. As far as an observer can be replaced by his measuring equipment this kind of realism might be compared with contextualistic realism in quantum mechanics.

As in quantum mechanics a contextualistic realist interpretation is a possible one, every observer having his own reality. Like in quantum mechanics, however, also here the empiricist solution to the interpretational problem may turn out to be the more natural one, relativistic quantities not being interpretable as properties of the object, but as pointer positions of measuring instruments. Relativity theory may be contrived such that it does not predict the length of an object relative to some observer, but such that it predicts the distance between the points an observer has marked off on his measuring rod corresponding to the interval he wants to measure. As in quantum mechanics the measurement procedure employed by the observer may introduce a certain nonideality, making it necessary to distinguish between measurement results obtained by means of different measurement procedures.

In order to make plausible that the measurement procedure may play an essential role also in relativity theory, we consider radial free fall into a Schwarzschild singularity. In the description[7] of the comoving observer (world line $r(\tau)$) the object is crossing the Schwarzschild radius in a finite time τ (its proper time); in the description of a stationary observer (world line $r(t)$) the object approaches $r = 2m$ asymptotically, its radial velocity approaching zero, without ever crossing the Schwarzschild radius (cf. fig. 3). On a realist interpretation of this phenomena reality with respect to the comoving observer is qualitatively different from the reality of the stationary observer. In the latter one's reality, if the falling object is a space ship, the rates of all processes on board are slowing down to a virtual standstill as the ship is approaching the Schwarzschild radius, bestowing any member of the space ship's crew with eternal life, be it a tortoise one. On the other hand, in the reality of the comoving observer the space ship will disappear into the singularity in a finite time.

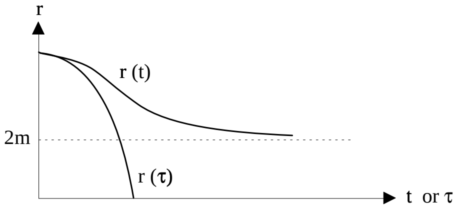

Figure 3: Radial free fall into a Schwarzschild singularity.

Although such a realist interpretation can be held without running into contradictions, it seems to me that not many physicists will be prepared to accept the nonexistence of an objective reality in which the space ship's fate is unambiguous (either it survives or it is wrecked). On a realist interpretation we would be confronted with such funny legal questions like the married state of a cosmonaut's wife (is she, or is she not a widow?). Not many physicists would advise to withhold her a widow's pension, because most of them will be convinced that the description by the comoving observer is closer to reality than the stationary observer's account.

Such a conviction undoubtedly stems from the idea that we should draw a distinction between reality and what we can observe about reality. If relativity theory would not describe reality as such, but only empirical evidence obtained by means of certain physical measurement processes, it seems possible to restore the idea of an objective reality, only observed in different ways by means of different observation methods. The difference between the two world lines of fig. 3 could be explained by realizing that information must be transmitted to the stationary observer, for instance, by means of light signals. Then, the slowing down of processes *as these are observed by the stationary observer* may be a peculiarity of the transmission process rather than a property of the object. Indeed, in the stationary frame of reference the radial velocity of light vanishes at the Schwarzschild radius, implying the transmission time between the object and the stationary observer to diverge as the object is at this point. Hence, if relativity theory would yield a description of the observer's observation rather than of reality itself, the difference between the two world lines of fig. 3 could be explained by the impossibility to transmit information on what is happening at the Schwarzschild radius to a stationary observer. Due to this circumstance the information contained in the stationary observer's description is not equivalent with the comoving observer's information: there is an irretrievable loss of information in the stationary observer's description, analogous to the loss of information obtaining in a noninvertible nonideal measurement in quantum mechanics.

As far as the region of space outside the Schwarzschild radius is concerned, the information of the two observers is equivalent because it is possible to calculate the corresponding part of one world line if the other one is known. This situation resembles the quantum mechanical situation of invertible nonideal measurements as exemplified by (6). The analogy may be pushed even further if it is possible to develop a partial ordering relation also for relativistic measurements. Measurements performed in a comoving inertial frame by means of rigid rods and geodesic clocks seem to be candidates for *maximal* relativistic measurements, any deviation from such a measurement scheme introducing a certain nonideality.

6. References

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